Determining the Hadron Spectrum Using Lattice QCD

Robert Edwards
Jefferson Lab

Twin Approaches to Confinement Physics
2012

Collaborators (Hadron Spectrum Collaboration):
J. Dudek, P. Guo, B. Joo, D. Richards (JLab), S. Wallace (Maryland)
N. Mathur (Tata), L. Liu, M. Peardon, S. Ryan, C. Thomas (Trinity College)
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Recent publications:
“Hybrid baryons”, in press PRD, 1201.2349
“Helicity operators for mesons in flight”, PRD85, 1107.1930
“Lightest hybrid meson supermultiplet”, PRD84, 1106.5515
“Excited state baryon spectroscopy”, PRD84, 1104.5152
“Isoscalar meson spectroscopy”, PRD83, 1102.4299
“Phase shift of isospin-2 scattering”, PRD83, 1011.6352
“Toward the excited meson spectrum”, PRD82, 1004.4930
“Highly excited and exotic meson spectrum”, PRL103, 0909.0200
Where are the “Missing” Baryon Resonances?

- What are collective modes?
- Is there “freezing” of degrees of freedom?
- What is the structure of the states?
- Where is the glue?
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Nucleon & Delta spectrum
PDG uncertainty on
B-W mass

QM predictions

Nucleon Mass Spectrum (Exp): 4°, 3°, 2°

Delta Mass Spectrum (Exp): 4°, 3°, 2°
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Spectrum from variational method

Two-point correlator

\[ C_{i,j}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle \]

\[ C_{i,j}(t) = \sum_n e^{-E_n t} \langle 0 | \Phi_i(0) | n \rangle \langle n | \Phi_j^\dagger(0) | 0 \rangle \]

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Matrix of correlators

\[
Z^n_i \equiv \langle n | \Phi^\dagger_i | 0 \rangle
\]

\[
C(t) = \begin{pmatrix}
\langle 0 | \Phi_1(t) \Phi_1^\dagger(0) | 0 \rangle & \langle 0 | \Phi_1(t) \Phi_2^\dagger(0) | 0 \rangle & \cdots \\
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“Rayleigh-Ritz method”

Diagonalize:

- eigenvalues \( \rightarrow \) spectrum
- eigenvectors \( \rightarrow \) spectral “overlaps” \( Z_i^n \)
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- eigenvectors → spectral “overlaps” \( Z_i^n \)

Each state optimal combination of \( \Phi_i \)

\[ \Omega^{(n)} = \sum_i \nu_i^{(n)} \Phi_i \]
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Benefit: orthogonality for near degenerate states
Baryon operators

Construction: permutations of 3 objects
Baryon operators

Construction: permutations of 3 objects

- Symmetric:
  - e.g., uud+udu+duu

- Antisymmetric:
  - e.g., uud-udu+duu-...

- Mixed: (antisymmetric & symmetric)
  - e.g., udu - duu & 2duu - udu - uud
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**Multiplication rules:**

- Symmetric × Antisymmetric → Antisymmetric
- Mixed × Mixed → Symmetric ⊕ Antisymmetric ⊕ Mixed...

1104.5152
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Multiplication rules:
• Symmetric $\rightarrow$ Antisymmetric
• Mixed x Mixed $\rightarrow$ Symmetric $\oplus$ Antisymmetric $\oplus$ Mixed
  • …

Color antisymmetric $\rightarrow$ Require Space $\times$ [Flavor $\times$ Spin] symmetric

Space: couple covariant derivatives onto single-site spinors - build any $J,M$

$\Phi^{JM} \leftarrow (CGC')_{i,j,k} \begin{bmatrix} \bar{D}_i \end{bmatrix} \begin{bmatrix} \bar{D}_j \end{bmatrix} [\Psi]_k$

$J \leftarrow 1 \otimes 1 \otimes S$

1104.5152
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Classify operators by permutation symmetries:
- Leads to rich structure

Multiplication rules:
- Symmetric \(\otimes\) Antisymmetric \(\rightarrow\) Antisymmetric
- Mixed x Mixed \(\rightarrow\) Symmetric \(\oplus\) Antisymmetric \(\oplus\) Mixed
- ...

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1104.5152
Baryon operator basis

All possible 3-quark operators up to two covariant derivatives: some $J^P$

$$\left( \left[ \text{Flavor } \otimes \text{ Dirac} \right] \otimes \text{Space}_{\text{symmetry}} \right)^{J^P}$$
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Spatial symmetry classification:

e.g., Nucleons: $N^{2S+1L_{\pi}} J^P$

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>#ops</th>
<th>E.g., spatial symmetries</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J=1/2^-$</td>
<td>32</td>
<td>$N^2P_M^{1/2-}$, $N^4P_M^{1/2-}$</td>
</tr>
<tr>
<td>$J=3/2^-$</td>
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<td>$N^2P_M^{3/2-}$, $N^4P_M^{3/2-}$</td>
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<tr>
<td>$J=5/2^-$</td>
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</tr>
<tr>
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<tr>
<td>$J=7/2^+$</td>
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- \( J=7/2^+ \): 4 \( N^{4D_{M} 7/2^+} \)

By far the largest operator basis ever used for such calculations

E.g., Nucleons: \( N^{2S+1L_{\pi}} J^P \)

Hold on, what are those \( P_M \)??
Operators featuring explicit glue

At two derivatives & \( L=1 \), have operators featuring a chromomagnetic B field

\[
\Phi \sim \varepsilon_{abc}(B_k \psi)_a \psi_b \psi_c + \ldots
\]

\[
B_k = \frac{1}{2} \varepsilon_{ijk}[D_i, D_j] \quad \text{where} \quad D_i = \partial_i - A_i
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Occurs only in Mixed spatial symmetry with \( L=1 \rightarrow P_M \) with positive parity

Hybrid operator: is 0 unless in a nontrivial background gauge field \( A \neq 0 \)
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Occurs only in \textit{Mixed} spatial symmetry with L=1 \to P_M with positive parity

\textit{Hybrid operator:} is 0 unless in a nontrivial background gauge field \( A \neq 0 \)

Note: Antisymmetric \( P_A \) operators not of this form. Is not 0 if \( A = 0 \)
Recall: two-point correlator matrix

\[ C_{ij}(t) = \langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \rangle \]
Just a basis

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Spectral decomposition

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Choose a random linear combination of operators

\[ \Phi'_i = \sum_j \eta_{ij} \Phi_j \]
Just a basis

Recall: two-point correlator matrix

\[ C_{ij}(t) = \left\langle 0 | \Phi_i(t) \Phi_j^\dagger(0) | 0 \right\rangle \]

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\[ C_{ij}(t) = \sum_n e^{-E_n t} \left\langle 0 | \Phi_i(0) | n \right\rangle \left\langle n | \Phi_j^\dagger(0) | 0 \right\rangle \]

Spectral “overlaps” \( Z_i^n \) change

\[ Z_i^n \equiv \left\langle n | \Phi_i^\dagger | 0 \right\rangle \]

Choose a random linear combination of operators

\[ \Phi_i' = \sum_j \eta_{ij} \Phi_j \]

Energies \( E_n \) unchanged
Spin identified Nucleon & Delta spectrum

Statistical errors    < 2%

arXiv:1104.5152, 1201.2349

$m_\pi \sim 396\text{MeV}$
Spin identified Nucleon & Delta spectrum

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$\sigma_{\pi} \sim 396 \text{MeV}$

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Spin identified Nucleon & Delta spectrum

Statistical errors < 2%

\[ m_\pi \sim 396\text{MeV} \]

SU(6) x O(3) counting
No parity doubling

[Graph showing the spectrum with masses and quantum numbers]
Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

$\rho \approx 396\text{MeV}$

arXiv:1104.5152, 1201.2349

$m_{\pi} \approx 396\text{MeV}$
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$m = 396\text{MeV}$
Spin identified Nucleon & Delta spectrum

Discern structure: spectral overlaps

$N^*$

$\Delta^*$

$N^*$

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Discern structure: spectral overlaps

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Discern structure: spectral overlaps

Significant mixing in $J^+$

$N^*$ and $\Delta^*$ levels:

$^{2S}_S$, $^{2S}_M$, $^{4S}_M$

$^{2D}_S$, $^{2D}_M$, $^{4D}_M$

$^{2P}_A$

13 levels/ops

$[56',0^+], [70,0^+], [56,2^+], [70,2^+], [20,1^+]$

$m_\pi = 396$ MeV
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Discern structure: spectral overlaps

Significant mixing in \( J^+ \)

\[ N^* \]
- \( \frac{1}{2}^+ \)
- \( \frac{3}{2}^+ \)
- \( \frac{5}{2}^+ \)
- \( \frac{7}{2}^+ \)

\[ \Delta^* \]
- \( \frac{1}{2}^+ \)
- \( \frac{3}{2}^+ \)
- \( \frac{5}{2}^+ \)
- \( \frac{7}{2}^+ \)

\[ N^* \]
- \( \frac{1}{2}^- \)
- \( \frac{3}{2}^- \)
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\[ \Delta^* \]
- \( \frac{1}{2}^- \)
- \( \frac{3}{2}^- \)
- \( \frac{5}{2}^- \)
- \( \frac{7}{2}^- \)

\( m / \text{GeV} \):
- 1.0
- 1.5
- 2.0
- 2.5
- 3.0

\( M \) levels/ops:
- \( ^2S_S \)
- \( ^2S_M \)
- \( ^4S_M \)
- \( ^2D_S \)
- \( ^2D_M \)
- \( ^4D_M \)
- \( ^2P_A \)

\( m_\pi = 396 \text{ MeV} \)
Spin identified Nucleon & Delta spectrum

No “freezing” of degrees of freedom

$N^*$

$\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, $\frac{7}{2}^+$

$\Delta^*$

$\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, $\frac{7}{2}^+$

13 levels/ops

$^{2S_S}{}^2S_M{}^4S_M$

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$^{2P_A}$

[56',0$^+$], [70,0$^+$], [56,2$^+$], [70,2$^+$], [20,1$^+$]

8 levels/ops

$^{2S_M}{}^4S_S$

$^{2D_M}{}^4D_S$

[56',0$^+$], [70,0$^+$], [56,2$^+$], [70,2$^+$]

$m/\text{GeV}$

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$m_\pi = 396\text{ MeV}
Hybrid baryons

Negative parity structure replicated: gluonic components (hybrid baryons)
Hybrid baryons

Negative parity structure replicated: gluonic components (hybrid baryons)

Predicted by Barnes & Close, 1983

See talk by J. Dudek

[70,1⁺]
P-wave

[70,1⁻]
P-wave

$m_\pi = 396$ MeV
Hadronic decays

Current spectrum calculations:
no evidence of multi-particle levels
Hadronic decays

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Plot the non-interacting meson levels as a guide

\[ |A(p)B(-p)\rangle \quad m_{AB} = \sqrt{m_A^2 + p^2} + \sqrt{m_B^2 + \bar{p}^2} \]
Hadronic decays

Current spectrum calculations: no evidence of multi-particle levels

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\[ |A(p)B(-p)\rangle \quad m_{AB} = \sqrt{m_A^2 + p^2} + \sqrt{m_B^2 + p^2} \]

Require multi-particle operators
- (lattice) helicity construction
- annihilation diagrams

\[ 1\,-- \quad 16^3 \quad 20^3 \]
Hadronic decays

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Require multi-particle operators
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- annihilation diagrams

Extract \( \delta(E) \) at discrete \( E \)
Spectrum of finite volume field

The idea: 1 dim quantum mechanics

Two spin-less bosons: \( \psi(x, y) = f(x-y) \rightarrow f(z) \)

\[
\begin{pmatrix}
-\frac{1}{m} \frac{d^2}{dz^2} + V(z)
\end{pmatrix} f(z) = E f(z)
\]

Solutions

\( f(z) \rightarrow \cos [k|z| + \delta(k)], \quad E = \frac{k^2}{m} \)

Quantization condition when \(-L/2 < z < L/2\)

\[ kL + 2\delta(k) = 0 \mod 2\pi \]

Same physics in 4 dim version (but messier)
Provable in a QFT (and relativistic)
Finite volume scattering

Scattering in a periodic cubic box (length $L$)
- Discrete energy levels in finite volume

E.g. just a single elastic resonance

At some $L$, have discrete excited energies

$E \rightarrow k; \quad kL + 2\delta(k) = 0 \mod 2\pi$

- $T$-matrix amplitudes $\rightarrow$ partial waves
- Finite volume energy levels $E(L) \leftrightarrow \delta(E)$
Resonances

Scattering of composite objects in non-perturbative field theory

\[ \text{Isospin} = 2 \pi \pi \]

\[ \delta_0(E) \& \delta_2(E) \]

\[ \ell = 2 \]

\[ \ell = 0 \]

1011.6352, 1203.????
Resonances

Scattering of composite objects in non-perturbative field theory

isospin=1 ππ

\[ \delta_0(E) \quad \& \quad \delta_2(E) \]

1011.6352, 1203.????

Feng, et al., 1011.5288
Resonances

Scattering of composite objects in non-perturbative field theory

Manifestation of “decay” in Euclidean space

Can extract pole position

Feng, et.al, 1011.5288
Resonances

Scattering of composite objects in non-perturbative field theory

Extracted coupling: stable in pion mass

\[ g_{\rho \pi \pi} \]

\[ m_{\pi}^2 \ (GeV^2) \]

Stability a generic feature of couplings??

Feng, et.al, 1011.5288
Some candidates: determine phase shift
Somewhat elastic

$S_{11} \rightarrow [N\pi]_S + [N\eta]_S$

$m_\pi = 396\text{ MeV}$
Hadronic decays – the full job

Need a lattice program of “amplitude analysis” - lots of room for help!

\[ m_{\pi} = 396 \text{ MeV} \]
Summary & prospects
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Results for baryon excited state spectrum:
• No “freezing” of degrees of freedom nor parity doubling
• Broadly consistent with non-relativistic quark model
• Extra bits interpreted as hybrid baryons
• Add multi-particle ops → baryon spectrum becomes denser
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Short-term plans: resonance determination!
• Lighter pion masses (230MeV available)
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Optimistic: see confluence of methods (an “amplitude analysis”)
• Develop techniques concurrently with decreasing pion mass