High Precision Measurement of the Proton Charge Radius
(C12-11-106)

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Outline

- Motivation of the experiment
- Proposed experiment
  - windowless hydrogen gas flow target
  - beam background
  - radiative corrections at very low $Q^2$
- Summary
**Motivation of the Experiment**

- Proton charge radius ($r_p$) is one of the fundamental quantities in physics
  - Important for nuclear physics:
    - long range structure of hadrons
    - test of upcoming lattice calculation
  - Critically important for atomic physics:
    - spectroscopy of atomic hydrogen
    - determination of Rydberg constant
      (the most accurately known constant in physics)
  - Connects nuclear and atomic physics
  - Arguably, the most referred quantity from outside of nuclear physics

- Three different ways to measure $r_p$
  - $ep \rightarrow ep$ elastic scattering at low $Q^2$
  - electronic-hydrogen spectroscopy (Lamb shift)
  - Muonic-hydrogen spectroscopy (Lamb shift)
More different analysis results than actual experiments

- Started with: $r_p \approx 0.81 \text{ fm}$ in 1963
- Reached to: $r_p \approx 0.88 \text{ fm}$ by 2006
Recent New Experimental Developments

  - Spectroscopic measurement with unprecedented error:
    \[
    \text{The result: } r_p = 0.84184(67) \text{ fm} \quad < 0.1\% \text{ total error}
    \]
    ✓ Different from most of previous experimental results !!!

- High statistics \(ep \rightarrow ep\) experiment at Mainz in 2010 (J. C.Bernauer, et al. PRL 105, 242001, 2010)
  - Relatively small \(Q^2\) range: \(Q^2 = [0.004 \rightarrow 1.0] \text{ (GeV/c)}^2\)
  - Statistical error \(\leq 0.2\%\)
  \[
  \text{The result: } r_p = 0.879(5)_{\text{stat}}(4)_{\text{sys}}(2)_{\text{mod}}(4)_{\text{group}}
  \]
  ✓ Confirms the previous results from \(ep\)-scattering;
  ✓ Consistent with CODATA06 value: \((r_p=0.8768(69) \text{ fm})\)
  ✓ No change in \(r_p\) average value !

- Plans for muonic-deuterium and muonic-helium Lamb shift measurements by same group
- New experimental proposal for \(\mu p \rightarrow \mu p\) scattering at PSI
Summary of Current $r_p$ Status

- **Open questions (after 2 years):**
  - additional corrections to muonic-hydrogen …? 
  - missing contributions to electronic-hydrogen …? 
  - higher moments in electric form factor …? 
  - different $e\Sigma$ and $\mu\Sigma$ interactions …? 
  - new physics beyond SM …? 

- many models, discussions … 
- no conclusions!

- **5 – 7 σ discrepancy between muonic and electronic measurements!**
  current “proton charge radius crisis”

- A novel high precision experiment performed with an independent method is needed to address this crisis.
The Proposed Experiment

- Proposed to PAC38 for high precision $ep \rightarrow ep$ scattering experiment:
  - with:
    - high resolution, large acceptance crystal calorimeter (HyCal)
      - non-magnetic-spectrometer method
      - simultaneous detection of Moller process
      - (best control of systematic errors)
    - reach smaller scattering angles: ($\theta = 0.8^0 - 3.8^0$)
      - $Q^2 = [2 \times 10^{-4} - 2 \times 10^{-2}]\ GeV^2$ first time for ep-experiments
      - essentially, model independent $r_\rho$ extraction
    - use high density windowless $H_2$ gas flow target
      - lowest background experiment
      - beam background fully under control with high quality CEBAF beam

- Two energies $E_0 = 1.1\ GeV$ and $2.2\ GeV$ to increase $Q^2$ range
- Will reach sub-percent precision
- Conditionally approved by PAC38 to finalize and address:
  - Full target design
  - Radiative corrections at very low $Q^2$
  - Full background simulations
In the limit of first Born approximation the elastic $ep$ scattering (one photon exchange):

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left( \frac{E'}{E} \right) \frac{1}{1 + \tau} \left( G_E^p Q^2 + \frac{\tau}{\epsilon} G_M^p Q^2 \right)
\]

\[
Q^2 = 4EE' \sin^2 \frac{\theta}{2} \quad \tau = \frac{Q^2}{4M_p^2} \quad \epsilon = \left[ 1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right]^{-1}
\]

Structure less proton:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \frac{\alpha^2 \left[ 1 - \beta^2 \sin^2 \frac{\theta}{2} \right]}{4k^2 \sin^4 \frac{\theta}{2}}
\]

At very low $Q^2$, cross section dominated by $G_{Ep}$:

\[
G_E^p Q^2 = 1 - \frac{Q^2}{6} \langle r^2 \rangle + \frac{Q^4}{120} \langle r^4 \rangle + \ldots
\]

r.m.s. charge radius given by the slope:

\[
\langle r^2 \rangle = -6 \left. \frac{dG_E^p(Q^2)}{dQ^2} \right|_{Q^2=0}
\]
Control of Systematic Errors

- Major improvements over previous experiments:
  1) Simultaneous detection of two processes
     - $ep \rightarrow ep$
     - $ee \rightarrow ee$ Moller scattering

- Windowless $H_2$ gas target

- Very low $Q^2$ range: $[2 \times 10^{-4} - 2 \times 10^{-2}]$ (GeV/c)$^2$

- Extracted yield for $ep \rightarrow ep$

\[
N_{exp}^{yield} (ep \rightarrow ep \text{ in } \theta_i \pm \Delta \theta) = \left(\frac{d\sigma}{d\Omega}\right)_{ep} (Q_i^2) \times N_{beam}^{H^1} \cdot N_{tgt}^{H} \cdot \varepsilon_{geom}^{ep} (\theta_i \pm \Delta \theta) \cdot \varepsilon_{det}^{ep}
\]

- ... and for $ee \rightarrow ee$, Moller

\[
N_{exp}^{yield} (\bar{e}^- e^- \rightarrow \bar{e}^- e^-) = \left(\frac{d\sigma}{d\Omega}\right)_{\bar{e}^- e^-} \times N_{beam}^{e^-} \cdot N_{tgt}^{H} \cdot \varepsilon_{geom}^{e^-} \cdot \varepsilon_{det}^{e^-}
\]

- Then, $ep$ cross section is related to Moller:

\[
\left(\frac{d\sigma}{d\Omega}\right)_{ep} (Q_i^2) = \left[\frac{N_{exp}^{yield} (ep \rightarrow ep \text{ in } \theta_i \pm \Delta \theta) \cdot \varepsilon_{geom}^{e^-} \cdot \varepsilon_{det}^{e^-}}{N_{exp}^{yield} (\bar{e}^- e^- \rightarrow \bar{e}^- e^-)} \cdot \varepsilon_{geom}^{ep} \cdot \varepsilon_{det}^{ep}\right] \left(\frac{d\sigma}{d\Omega}\right)_{\bar{e}^- e^-}
\]

- Two major sources of systematic errors, $N_e$ and $N_{tgt}$, typical for all previous experiments, cancel out.

- Moller scattering will be detected in coincident mode in HyCal acceptance
- High resolution, large acceptance HyCal calorimeter
  \((\text{PbWO}_4\ \text{part only})\)
- Windowless \(\text{H}_2\) gas flow target
- \(XY\) – veto counters
- Vacuum box, one thin window at HyCal only
Windowless $\text{H}_2$ Gas Flow Target

- cell length: 4.0 cm
- cell diameter: 8.0 mm
- cell material: 30 μm Kapton
- input gas temp.: 25 K
- target thickness: $1 \times 10^{18}$ H/cm$^2$
- average density: $2.5 \times 10^{17}$ H/cm$^2$
- gas mass-flow rate: 6.3 Torr-l/s

Pre-engineering design finalized
NSF MRI proposal developed and submitted for target construction
Beam Background Simulations

- GEANT based MC code developed with realistic experimental setup, including current windowless H₂ target

- **Beam test** successfully performed in Hall B
  - Thanks to Hall B management, staff and Accelerator group

- high quality CEBAF beam parameters:
  - Signal/Noise $> 10^7$
  - beam size at target: $\sim 100 \ \mu m$

- Target design optimized to minimize beam background

- **Beam background** estimated to be at percent level
  - Major contribution from 30 $\mu m$ Kapton cell

- periodic “empty target” measurements to control background on sub-percent level.
- Use Bardin-Shumeiko covariant formalism to calculate RC
- Beyond the ultra relativistic approx. mass of the electron is not neglected
- The change in the cross section is less than 0.2% at the lowest $Q^2$ point
- Modified the elastic $ep$ scattering codes ELRADGEN and MERADGEN accordingly
Radiative Corrections (cont’d)

Corrections to the cross sections

\(ep: \sim 8\text{-}13\%\)
\((\text{ELRADGEN})\)

Möller: \(\sim 2\text{-}3\%\)
\((\text{MERADGEN})\)
Extraction of Proton Charge Radius

- Extraction of $r_p$ from MC simulations with and without radiation
- Estimated systematic uncertainty $< 0.3\%$
Responses to Theory Comments

- Theory comment:
  1) “…The Coulomb corrections should be re-discussed (they were in the original proposal) to convincingly show they will cause no problems for the data analysis. … ”

   ✓ full Coulomb simulations performed for our kinematics (Fig. right)
   ✓ compared with other modern calculations (Fig. left).
   ✓ Coulomb corrections for our $Q^2$ range and $\varepsilon \approx 1$ are smaller than the sensitivity of this experiment.

   - J. Arrington, PRL 107, 119101, 2011
   - J.C. Bernauer, et al. PRL 107, 119102, 2011
Responses to TAC Comments

- TAC comments:

  1) “…coordinate with JLab engineers during the design and construction of the target to ensure that it meets the lab’s stringent safety requirements …”

    We agree with this comment and already from the pre-engineering design phase of the target we have closely worked with Jlab engineers. We will continue this during the entire period of the full engineering design, construction and installation of the target.

  2) “…A plan should be devised of how the focal plane will be maintained and calibrated after the Hall upgrade to 12 GeV operation …”

    The photon tagger will be used for the
    (a) gain equalizing to make an effective trigger and
    (b) energy calibration of HyCal.

    For this, only a small part (upper ~20%) of the focal plane is needed.
    We will continue discussions and work out all possible tagger related options with Hall B management.
Beam Time Request and Error Budget

- target thickness: $N_{tgt} = 1 \times 10^{18}$ H atoms/cm$^2$
  $I_e : \sim 10$nA  ($N_e = 6.25 \times 10^{10}$ e$^-$/s)

- for $E_0 = 1.1$ GeV, Total rate for $ep \rightarrow ep$
  $N_{ep} = N_e \times N_{tgt} \times \Delta \sigma \times \varepsilon_{geom} \times \varepsilon_{det}$
  $\approx 150$ events/s $\approx 12.8$ M events/day

Rates are high, however, for 0.5% stat. error for the last $Q^2 = 5 \times 10^{-3}$ (GeV/c)$^2$ bin, 2 days are needed

### Contributions

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<th>Estimated Error (%)</th>
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<tr>
<td>Statistical error</td>
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<tr>
<td>Acceptance (including $Q^2$ determination)</td>
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<td>Detection efficiency</td>
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<td>Radiative corrections</td>
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<td>Background and PID</td>
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<td>Fitting error</td>
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- Beam time

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<tr>
<td>$H_2$ gas target commission</td>
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<td>Statistics at 1.1 GeV</td>
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<tr>
<td>Energy change</td>
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<td>Statistics at 2.2 GeV</td>
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<td>Empty target runs</td>
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</tr>
<tr>
<td>Total</td>
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</table>

- Estimated error budget (added quadratically)
Summary

- A novel experiment for the proton size measurement with an independent method is required to address the current “proton charge radius crisis”. Jlab is in a position to make a long lasting impact on this important quantity in a timely and unique way.

- New magnetic-spectrometer-free experiment with tight control of systematic errors:
  - ep→ep cross sections normalized to Moller scattering
  - reach very low Q^2 range: [2×10^{-4} – 2×10^{-2}] GeV^2
  - windowless hydrogen gas flow target

- Questions raised by PAC38 addressed:
  - Pre-engineering design of the new target is completed
  - Radiative correction codes improved at this Q^2 to provide less than 0.3% uncertainty
  - Full Monte Carlo simulation code developed for the experiment. Backgrounds are at percent level

- Requesting 15 days of beam time to measure r_p with sub-percent precision
The End
Muonic Hydrogen Experiment (2010)

- Muonic hydrogen Lamb shift experiment at PSI
- \( r_p = 0.84184(67) \) fm \( \Rightarrow \) Unprecedented less than 0.1% precision
- Different from most of previous experimental results and analysis
Recent Mainz High Precision $ep \rightarrow ep$ Experiment

- Large amount of overlapping data sets
- Statistical error $\leq 0.2\%$
- Luminosity monitoring with spectrometer
- Additional beam current measurements
- $Q^2 = [0.004 – 1.0] \ (GeV/c)^2$ range

Many form factor models, fit to all cross sections.

The result:

$$ r_p = 0.879(5)_{\text{stat}}(4)_{\text{sys}}(2)_{\text{mod}}(4)_{\text{group}} $$

- Confirms the previous results from $ep \rightarrow ep$ scattering;
- Consistent with CODATA06 value: $(r_p=0.8768(69) \text{ fm})$
- No change in $r_p$ average value!

J. Bernauer, PRL 105, 242001, 2010
Control of Systematic Errors (Calorimeter Misalignment)

- accuracy of engineering survey: 0.7 mm
- Off-line check with co-planarity of Moller events (done in PrimEx experiments with Compton)
  - HyCal misalignment is not a problem for $r_p$ extraction
Elastic/Moller Overlap

- Overlap of $E_e$ spectra of radiated events
Elastic/Moller Overlap

- Overlap of $E_e$ spectra of radiated events contamination from Moller events (for $0.8 < \theta_e < 3.8$ deg)
Will analyze Moller events in 3 different ways:

1) Single-arm method: one Moller $e^-$ is in the same $Q^2$ range

$\varepsilon_{\text{det}}$ will be measured for [0.5 – 2.0] GeV range

Relative $\varepsilon_{\text{det}}$ are needed for this experiment

$$\frac{d\sigma}{d\Omega}_{ep} (Q_i^2) = \left[ \frac{N_{\text{exp}}^{\text{yield}}(ep \rightarrow ep \text{ in } \theta_i \pm \Delta \theta)}{N_{\text{exp}}^{\text{yield}}(e^-e^- \rightarrow e^-e^-)} \right] \frac{d\sigma}{d\Omega}_{e^-e^-}$$

2) Coincident Method

$$\frac{d\sigma}{d\Omega}_{ep} (Q_i^2) = \left[ \frac{N_{\text{exp}}^{\text{yield}}(ep \rightarrow ep \text{ in } \theta_i \pm \Delta \theta)}{N_{\text{exp}}^{\text{yield}}(e^-e^- \rightarrow e^-e^-)} \cdot \frac{\varepsilon_{\text{det}}}{\varepsilon_{\text{geom}}} \cdot \frac{\varepsilon_{\text{geom}}^{\text{all PWO}}}{\varepsilon_{\text{geom}}^{\text{on PWO}}} \right] \frac{d\sigma}{d\Omega}_{e^-e^-}$$

3) Integrated over HyCal acceptance

$$\frac{d\sigma}{d\Omega}_{ep} (Q_i^2) = \left[ \frac{N_{\text{exp}}^{\text{yield}}(ep, \theta_i \pm \Delta \theta)}{N_{\text{exp}}^{\text{yield}}(e^-e^-, \text{ on PWO})} \cdot \frac{\varepsilon_{\text{det}}^{\text{all PWO}}}{\varepsilon_{\text{geom}}^{\text{all PWO}}} \cdot \frac{\varepsilon_{\text{geom}}^{\text{on PWO}}}{\varepsilon_{\text{geom}}^{\text{on PWO}}} \right] \frac{d\sigma}{d\Omega}_{e^-e^-}$$

Relative $\varepsilon_{\text{det}}$ will be measured with high precision.

Contribution of $\varepsilon_{\text{det}}$ and $\varepsilon_{\text{geom}}$ in cross sections will be on second order only.
Event Rate and Statistics

With hydrogen gas target thickness: \( N_{\text{tgt}} = 1 \times 10^{18} \) H atoms/cm\(^2\)
Electron beam intensity: \( \sim 10 \) nA \( (N_e = 6.25 \times 10^{10} \) e\(^-\)/s) \n
- For \( E_0 = 1.1 \) GeV run
  - Total rate for \( ep \rightarrow ep \)
    \[ N_{ep} = N_e \times N_{\text{tgt}} \times \Delta \sigma \times \varepsilon_{\text{geom}} \times \varepsilon_{\text{det}} \]
    \[ = 6.25 \times 10^{10} \times 1.10^{18} \times 3.14 \times 10^{-26} \times 0.75 \times 1. \]
    \[ \approx 150 \text{ events/s} \]
    \[ \approx 12.8 \text{ M events/day} \]
  - Rates are high, however, for 0.5% stat. error for the last \( Q^2 = 5 \times 10^{-3} \) (GeV/c)^2 bin, 2 days are needed

- Rate for \( ee \rightarrow ee \) cross sections are higher, but geometrical acceptance is less:
  \[ N_{ee} = 6.25 \times 10^{10} \times 1.10^{18} \times 6.8 \times 10^{-26} \times 0.005 \times 1. \]
  \[ \approx 200 \text{ events/s} \]
  \[ \approx 17.3 \text{ M events/day} \]
  High rate will provide good statistics

- For \( E_0 = 2.2 \) GeV run:
  - The \( ee \rightarrow ee \)
    \[ \sigma_{ee} \approx 1/E_0 \]
    But, \( \varepsilon_{\text{geom}} \) is increasing, the rate is \( \approx \) constant
  - The \( ep \rightarrow ep \)
    \[ \sigma_{ep} \approx 1/E_0^2 \]
    However, only last bin: \( Q^2 = 2 \times 10^{-2} \) (GeV/c)^2 will have \( \approx 1\% \) stat. error for the same 2 days of run
- Signal/Noise $\approx 10^8$ level reached starting from $\pm 2$ mm from beam center
- Electron beam size $\approx 25 \, \mu\text{m}$