Nitrogen Elastic Form Factors and Elastic Tail

2/10/16
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Background

• Thanks to Vince for sending me the updated saGDH nitrogen cross sections
• And also the simulation code for collimator and finite acceptance effects for the elastic tail
  – Vince sent me the elastic tail using the code for two settings and I was able to reproduce his results using the same code.
  – Vince’s tech-note on the codes:
    • Had to make minor modifications to code described here and linked to in the tech-note to re-include nitrogen capability.

• PROBLEM:
  – After doing the tail subtraction and running the inelastic RC code the fully RC’d cross sections go negative at high nu (these are unpolarized XS’s...)
    • 2135 MeV/6degrees @ about 1000 MeV in nu
    • Also occurs at 2845 MeV/6 degrees
  – Assume the cause is related to the fact that elastic tail and measured cross section are on the same order of magnitude at high nu
    • At the very end of the tail: tail and measured XS (with tail) only differ by ~30%.
Saw the same problem with the original saGDH nitrogen cross sections so don’t believe it’s a problem cross sections themselves
  – Or the finite acceptance/punchthrough corrections
  – Previously I had just cut off the cross sections after they went negative

That leaves either:
  – the nitrogen elastic form factors
  – the inelastic interpolation
  – Or the elastic tail formalism itself

First tried varying the cross section used in the interpolation for the inelastic RCs
  – For 2135 MeV there is no lower energy cross section, so I have to use a model (Bosted) for it
  – Tried varying the overall scale and also the beam energy and nu range of this cross section
  – Had a small effect on the resultant RC’d cross section (could move the zero crossing buy a handful of Mev)
  – Not large enough to be the culprit

Moving onto the nitrogen elastic form factors...
Nitrogen Elastic Form Factors

• Nitrogen Elastic Form Factor fit comes from:
  – Fit/Data covers a range from $0.028 \text{ GeV}^2 < Q^2 < 0.28 \text{ GeV}^2$

• Lowest $Q^2$ called in tails is
  – $\sim 1e-4 \text{ GeV}^2$ for internal tail and $0.01 \text{ GeV}^2$ for external (and internal in peaking approx.)
  – Potential source of problem

<table>
<thead>
<tr>
<th>$\theta$ (deg)</th>
<th>$q$ (fm$^{-1}$)</th>
<th>$10^4 \times F^2(q^2)$ (experimental)</th>
<th>$10^4 \times F^2(q^2)$ (calculated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.86</td>
<td>1643 ± 63</td>
<td>1840</td>
</tr>
<tr>
<td>50</td>
<td>1.07</td>
<td>637 ± 16</td>
<td>679</td>
</tr>
<tr>
<td>60</td>
<td>1.26</td>
<td>195 ± 6</td>
<td>209</td>
</tr>
<tr>
<td>70</td>
<td>1.45</td>
<td>42.3 ± 1.2</td>
<td>44.4</td>
</tr>
<tr>
<td>80</td>
<td>1.62</td>
<td>7.25 ± 0.54</td>
<td>7.84</td>
</tr>
<tr>
<td>90</td>
<td>1.78</td>
<td>3.50 ± 0.31</td>
<td>3.34</td>
</tr>
</tbody>
</table>

Table 3.9: Elastic form factors of $^{14}\text{N}$ at $E_{\text{beam}} = 250 \text{ MeV}$. 

Taken from P. Djawotho thesis, but I confirmed the trend. Fit is worse at lower $Q^2$.
ReFit and New Data

- A few weeks ago I showed slides where I did a refit of the few low $Q^2$ points using a charge form factor fit.
- Calculated all tails using this charge form factor fit and then looked at the ratio between the inelastically radiated Bosted model and the data with the tail subtracted.

Would expect this ratio to level off because there isn’t any physics out here.

Still a problem with the tail?
n2: saGDH 2845 MeV

\[ \frac{d^2\sigma}{d\Omega d\nu} (\text{nb/MeV/Sr}) \]

- \( \text{saGDH XS} \)
- \( \text{saGDH Tail Sub} \)
- \( \text{Elastic Tail} \)

\[ 0 \leq \nu \leq 2000 \]

\[ 10^0 \leq \frac{d^2\sigma}{d\Omega d\nu} \leq 10^5 \]

\[ 1.8 \leq \text{Ratio} \leq 1.0 \]

\[ 0 \leq \nu \leq 2000 \]
n2: saGDH 2234 MeV

![Graph showing data for n2: saGDH 2234 MeV with markers and lines indicating saGDH XS and saGDH Tail Sub, along with Elastic Tail.]
New Data

• Maybe the fit is still wrong? Need to find more to data to see if this is the case
• Found four additional N2 data sets (after much searching)
• One more data set out there but I haven’t be able to get it:

Relative to 1H cross section
New Data

\[ F^2(Q^2) \]

- Dally et al. (1970)
- Schutz (1975)
- Bentz (1971)
- Bishop et al. (1964)
- Voitsekhovskii et al. (1978)

- ReFit of Dally Function
- Original Dally Fit
- 2 Parameter Fermi-Function Fit
Going Forward

• 2 Parameter Fermi-Function fit under-predicted elastic cross section (and thus elastic tail) at low $Q^2$
• Have new fit that is an update to the fit parameters of the original Dally fit
  – Still need to do a quick check to see how the tails from the two fits compare
• Large tail at large energy loss dominated by contributions from radiation length/radiative energy loss before scattering
  – Perhaps inaccurate assumptions/formulation of the tail in this region
  – Made a small mod. to Vince’s simulation to change input rad length based upon scattering vertex
    • Had an effect but not big enough
• Going to also check inelastic radiation and see how incoming rad length effects this
• Otherwise cut off cross sections where the start to tail off (for now)
• Inelastic code all set up and ready to go (complete with systematic error)
• Suggestions? Comments?
BACKUP
Dally Fit

\[ F^2(q^2, \theta) = F_L^2(q^2) + (\frac{1}{2} + \tan^2 \frac{1}{2} \theta) F_T^2(q^2) \]

Coulomb Term

\[ F_L^2(q^2) = \sum_{\lambda=0}^{\infty} F_C \lambda^2(q^2) \]

\[ |F_C(q^2)|^2 = \left| \left(1 - \frac{2}{3} \frac{Z-2}{Z} x\right) e^{-i\phi} \right|^2 \]

\[ F_C^2(q^2) = \frac{q^4}{180} \frac{(J+1)(2J+3)}{J(2J-1)} \left(\frac{Q}{Z}\right)^2 e^{-2(x+d)} \]

Transverse Term

Selection rules determine the non-zero contributions

\[ F_T^2(q^2) = \sum_{\lambda=1}^{\infty} \left[ F_E \lambda^2(q^2) + F_M \lambda^2(q^2) \right]. \]

\[ F_M^2(q^2) = \frac{2}{3} \frac{J+1}{J} \frac{x}{a^2m_p^2} \left(\frac{\mu}{Z}\right)^2 \left[1 - \frac{2}{3} \alpha + \alpha \frac{2}{3} \alpha^2 \right] e^{-2(x+d)} \]

Results from the Dally Fit

Fitted terms are \( \mu/Q \) and related to \( x \) and \( d \) (see side bar)

| Nucleus | Energy (MeV) | \( \alpha \) (F) | \( |\mu| \) (\( \mu_n \)) | \( Q \times 10^{24} \) (cm\(^2\)) | \( \langle r \rangle \) (F) | Degrees of freedom | \( \mu \) static value (\( \mu_n \)) | \( Q \times 10^{24} \) static value (cm\(^2\)) |
|---------|--------------|-----------------|----------------------|-------------------------------|------------------|----------------|----------------|--------------------------|
| \( \text{^14}_N \) | 400 | 1.75 | 30 | 0.44 | (1.52 \( \pm 4.2 \)) \( \times 10^{-5} \) | 2.64 | 38 | 10 | 0.4 | 2 \( \times 10^{-2} a \) |
| | | \( \pm 1.2 \) | \( \pm 0.35 \) | | | | | | | | \( 7.1 \times 10^{-2} b \) |
| \( \text{^15}_N \) | 250 | 1.74 | 2.5 | -0.64 | 0 | 2.65 | 42 | 14 | 0.28 | 0 |
| | | \( \pm 0.2 \) | \( \pm 0.4 \) | | | | | | | | |
| | 400 | 1.72 | 4.5 | -0.24 | 0 | 2.65 | 34 | 13 | 0.28 | 0 |
| | | \( \pm 0.14 \) | \( \pm 0.13 \) | | | | | | | | |
Dally Fit: Relative Contributions

Two options:
1. Refit existing data with same form as Dally to see if fit can be improved
   - Tried this and still a poor fit at low momentum transfer
2. Do a charge form factor fit only and just to the and restrict it to the lowest few momentum transfer points

Low momentum transfer dominated by C0 (charge) term

FIG. 9. The fit to the elastic scattering from N$^{14}$ at 400 MeV using the shell-model description. The monopole (C0), magnetic dipole (M1), and electric quadru-pole (C2) term contributions are shown in dashed lines. The solid curve is the total result of three terms.
Charge Form Factor Fit

• Reference paper for charge-density distribution:
• For a spherically symmetric charge density the charge form factor is (see Particles and Nuclei, Povh, Rith, Scholz and Zetsche):

\[
F(q^2) = 4\pi \int_0^\infty \rho(r) \frac{\sin(qr)}{qr} r^2 dr
\]

• Choice of the charge density is model dependent
• I considered two different models:
  – Harmonic Oscillator: \( \rho(r) = \rho_0(1 + \alpha(r/a)^2)\exp(-(r/a)^2) \)
  – 2 Parameter Fermi Model:
    \[
    \rho(r) = \rho_0/(1 + \exp((r - \xi)/\zeta))
    \]

Fermi Model

- Ended up choosing Fermi model because it was a better fit to the data
- Because only a charge form factor fit, should only be used for low $Q^2$ data points
  - Ultimately looking for best agreement at lowest momentum transfer squared

\[ Q^2 < 0.05 \text{ GeV}^2 \]

\[ c = 2.043 \pm 0.0632 \]
\[ z = 0.600 \pm 0.0141 \]

\[ Q^2 < 0.10 \text{ GeV}^2 \]

\[ c = 2.291 \pm 0.0158 \]
\[ z = 0.538 \pm 0.0071 \]
Fermi Model Cont’d

- Comparison of the two Fermi model fits

\( Q^2 < 0.05 \text{ GeV}^2 \)

<table>
<thead>
<tr>
<th>Q2</th>
<th>F2 Measure</th>
<th>F2 Fit Calc</th>
<th>Ratio</th>
</tr>
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<tbody>
<tr>
<td>0.029</td>
<td>0.1635</td>
<td>0.1630</td>
<td>1.003</td>
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<tr>
<td>0.044</td>
<td>0.0637</td>
<td>0.640</td>
<td>0.995</td>
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<tr>
<td>0.048</td>
<td>0.0500</td>
<td>0.0498</td>
<td>1.004</td>
</tr>
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\( Q^2 < 0.10 \text{ GeV}^2 \)

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<td>0.0637</td>
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<td>0.048</td>
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<td>0.058</td>
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<td>0.062</td>
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<td>0.067</td>
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<td>0.944</td>
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<td>0.074</td>
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<td>0.980</td>
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<td>0.081</td>
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<tr>
<td>0.085</td>
<td>0.0028</td>
<td>0.0029</td>
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<tr>
<td>0.093</td>
<td>0.00149</td>
<td>0.00142</td>
<td>1.046</td>
</tr>
</tbody>
</table>

- Obviously smaller range in \( Q^2 \) fit gives a better fit at low \( Q^2 \)
Note on Elastic Tails

• When calculating tails only use updated fit below the fitted $Q^2$ limit
  – Otherwise use the standard Dally fit
  – And only use it at effected energy settings

• Because only doing a fit to charge form factor have to use peaking approximation in the calculation of the internal elastic tail
  – On the plus side it vastly speeds up Vince’s simulation on the tails (a few+ hours becomes a few minutes)

• Using either Fermi fit (0.05 or 0.1) produces positive cross sections after inelastic RC’s !!
400 MeV, Djawotho thesis

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<td>1.12</td>
<td>500 ± 15</td>
<td>496</td>
</tr>
<tr>
<td>35</td>
<td>1.22</td>
<td>261 ± 7</td>
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<tr>
<td>38</td>
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<td>120 ± 4</td>
<td>126</td>
</tr>
<tr>
<td>40</td>
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<td>73.2 ± 2.3</td>
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<tr>
<td>43</td>
<td>1.48</td>
<td>28.8 ± 0.9</td>
<td>30.0</td>
</tr>
<tr>
<td>45</td>
<td>1.55</td>
<td>14.7 ± 0.5</td>
<td>14.4</td>
</tr>
<tr>
<td>48</td>
<td>1.64</td>
<td>6.18 ± 0.19</td>
<td>5.67</td>
</tr>
<tr>
<td>50</td>
<td>1.70</td>
<td>3.56 ± 0.10</td>
<td>3.72</td>
</tr>
<tr>
<td>53</td>
<td>1.80</td>
<td>3.03 ± 0.13</td>
<td>3.33</td>
</tr>
<tr>
<td>55</td>
<td>1.86</td>
<td>3.32 ± 0.11</td>
<td>3.67</td>
</tr>
<tr>
<td>65</td>
<td>2.16</td>
<td>3.43 ± 0.15</td>
<td>3.39</td>
</tr>
<tr>
<td>75</td>
<td>2.44</td>
<td>1.72 ± 0.11</td>
<td>1.34</td>
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<tr>
<td>80</td>
<td>2.58</td>
<td>0.799 ± 0.076</td>
<td>0.686</td>
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<tr>
<td>85</td>
<td>2.70</td>
<td>0.310 ± 0.060</td>
<td>0.354</td>
</tr>
</tbody>
</table>

Table 3.10: Elastic form factors of $^{14}$N at $E_{\text{beam}} = 400$ MeV.