Inelastic RC Systematic Error

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Overview

- Want to do the same thing I did for the elastic tail and check the error with the approximations and calculations done in inelastic RCs
- Some errors carry over from the elastic tail so I don't have to redo them:
  - Fbar (higher order virtual photon corrections)
  - Energy-peaking approximation used in evaluating external corrections
  - Born approximation
- New potential sources of error
  - Angle-peaking approximation in internal bremsstrahlung
  - Choice of straggling function for external bremsstrahlung
    - Did check this for the elastic tail but should do it again
  - Soft photon correction factor
  - Interpolation/extrapolation error
    - Including using a model as source for lowest energy setting to RADCOR input

Internal Bremsstrahlung

- For the elastic tail, the full expression for the internal bremsstrahlung is

$$
\begin{aligned}
\sigma_{\text{exact}} = \left(\frac{d^2\sigma}{d\Omega dE_p}\right)_{\text{ex}} &= \frac{\alpha^3}{2\pi}\left(\frac{E_p}{E_s}\right)\int_{-1}^{1}\frac{2M_T\omega d(\cos\theta_k)}{q^4(u_0-|\vec{u}|\cos\theta_k)} \\
&\times\left(\tilde{W}_2(q^2)\left\{\frac{-am^2}{x^3}\left[2E_s(E_p+\omega)+\frac{q^2}{2}\right]-\frac{a'm^2}{y^3}\left[2E_p(E_s+\omega)+\frac{q^2}{2}\right]\right.\right. \\
&-2+2\nu(x^{-1}-y^{-1})\{m^2(s\cdot p-\omega^2)+(s\cdot p)[2E_sE_p-(s\cdot p)+\omega(E_s-E_p)]\} \\
&+x^{-1}\left[2(E_sE_p+E_s\omega+E_p^2)+\frac{q^2}{2}-(s\cdot p)-m^2\right] \\
&\left.-y^{-1}\left[2(E_pE_s+E_p\omega+E_s^2)+\frac{q^2}{2}-(s\cdot p)-m^2\right]\right\} \\
&+\tilde{W}_1(q^2)\left[\left(\frac{a}{x^3}+\frac{a'}{y^3}\right)m^2(2m^2+q^2)+4+4\nu(x^{-1}-y^{-1})(s\cdot p)(s\cdot p-2m^2)\right. \\
&\left.\left.+(x^{-1}-y^{-1})(2s\cdot p+2m^2-q^2)\right]\right),
\end{aligned}
$$

Angle-Peaking Apprximation

- Full integral is calculated at all possible photon emission angles
  - But in reality, the majority of photon's are emitted in the same direction as the incoming and outgoing electrons
- Can then approximate full integral as
  - Uses an 'equivalent radiator' == internal bremsstrahlung approximated as an external bremsstrahlung correction

$$\sigma_{\rm int}^{\rm pk}(E_s, E_p) = \frac{bt_r \phi(v_p)}{\omega_p} \tilde{F}(q_p^2)\sigma_{\rm el}(E_s) + \frac{M_T + 2(E_s - \omega_s)\sin^2\frac{\theta}{2}}{M_T - 2E_p\sin^2\frac{\theta}{2}} \frac{bt_r\phi(v_s)}{\omega_s} \tilde{F}(q_s^2)\sigma_{\rm el}(E_s - w_s),$$

$$\omega_s = E_s - \frac{E_p}{1 - 2\frac{E_p}{M_T}\sin^2\frac{\theta}{2}},$$

$$\omega_p = \frac{E_s}{1 + 2\frac{E_s}{M_T}\sin^2\frac{\theta}{2}} - E_p,$$

$$bt_r = \frac{\alpha}{\pi}\left[\ln\left(\frac{q^2}{m_e^2}\right) - 1\right]$$

Energy of incoming (outgoing) photon

Follows Eqs (B.6) – (B.11) of Mo and Tsai, Rev. Mod. Phys 41 205, (1969)

Inelastic Internal Bremsstrahlung

- Inelastic isn't limited to $W = M_T$ so the inelastic internal contribution is the elastic internal tail integrated over all possible M_f

$$\frac{d\sigma}{d\Omega dE}(\omega > \Delta) = \frac{\alpha^3}{2\pi}\frac{E_p}{E_s M_T}\int_{-1}^{1} d(\cos\theta_k)\int_{\Delta}^{\omega_{\max(\cos\theta_k)}}\frac{\omega d\omega}{q^4} B_{\mu\nu}T^{\mu\nu}$$

Contains the inelastic structure functions

- Δ parameter avoids a divergence at $\omega = 0$, so the full internally inelastically radiated spectrum is

$$\frac{d\sigma_r}{d\Omega dE}(E_s, E_p) = e^{-\delta_r(\Delta)}\tilde{F}(q^2)\frac{d\sigma}{d\Omega dE} + \frac{d\sigma_r}{d\Omega dE}(\omega > \Delta)$$

  - This is a slightly tweaked version of (B.6) of MT, but it keeps choice of fbar consistent across calculations (See B. Badalek *et al*. arxiv:hep-ph/940328v1 (1994).)

Inelastic Peaking Approximation

- The equivalent correction in the angle-peaking approximation is
  - Dropping the soft-photon terms. This is SLAC-PUB-848 (1971) (C.23)

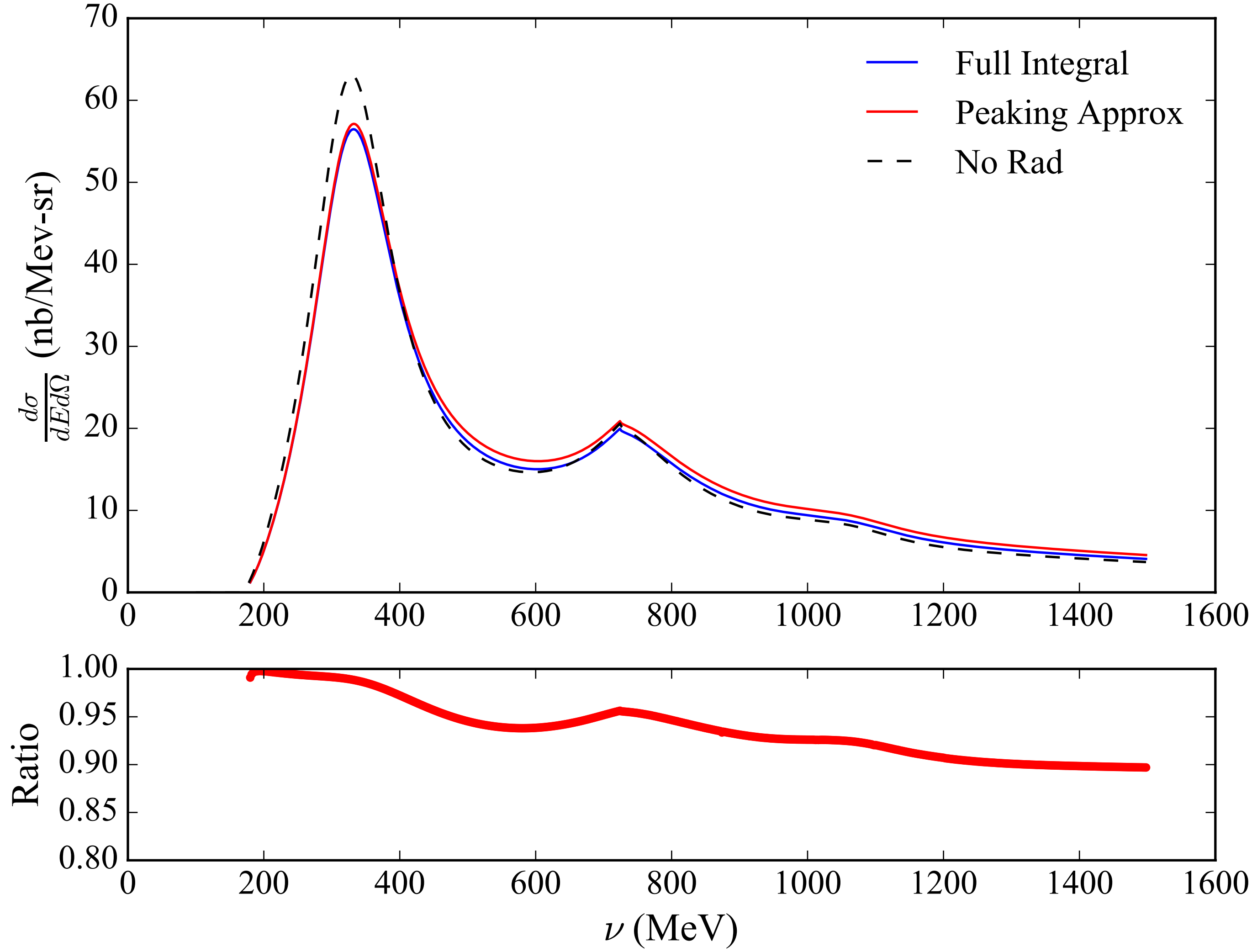
$$\frac{d\sigma_r}{d\Omega dE}(E_s, E_p) = \tilde{F}(q^2)\Bigg[\left(\frac{R\Delta}{E_s}\right)\left(\frac{\Delta}{E_p}\right)^{bt_r}\frac{d\sigma}{d\Omega dE}(E_s, E_p)$$
$$+ \int_{E_p+\Delta}^{E_{p\max}} dE'_p \frac{d\sigma}{d\Omega dE}(E_s, E'_p)\frac{bt_r}{2(E'_p - E_p)}\phi(\frac{E'_p - E_p}{E'_p})$$
$$+ \int_{E_{s\min}}^{E_s - R\Delta} dE'_s \frac{d\sigma}{d\Omega dE}(E'_s, E_p)\frac{bt_r}{2(E_s - E'_s)}\phi(\frac{E_s - E'_s}{E_s})\Bigg]$$

- Angle peaking approximation is used because:
  - Significantly faster (<1 min to run compared to multiple hours for full integral)
  - In general the inelastic structure functions aren't known but the total cross section is, so this can be used for unfolding!

Comparing the Two

Using the Bosted model

p: 2135 Mev/ 6 degrees



Going Forward

- Difference between the two calculations is 5-10% for the proton
  - Haven't checked yet to see how much internal radiation contributes to total radiated cross section
- Need to make the switch to nitrogen for the saGDH data
- Machinery is also in place to check model input systematic
- Already looked at the systematic for interpolation
  - Previously showed a model radiated and then unfolded.
- Move on to the other systematic checks I mentioned