Overview

- Walk through results section of my thesis
- Try to highlight what I haven’t talked about before at weekly meetings

**Kinematic Coverage**
Intensity of points is proportional to number of events we took
2.5 T scaled for target polarization difference
Asymmetries

• Correct stat. error bars in asym for fluctuation due to pre-scale factor:
  – [https://hallaweb.jlab.org/dvcslog/g2p/170](https://hallaweb.jlab.org/dvcslog/g2p/170)

\[
S = \sqrt{1 - LT \cdot f_A \left( 1 - \frac{1}{p_s} \right)} \quad \delta A \approx \frac{1}{2} \sqrt{\frac{S_+^2}{N_+} + \frac{S_-^2}{N_-}}
\]

• Form final asymmetries as stat weighted average according to:

\[
A = \frac{\sum_i A_i / \delta A_i^2}{\sum_i 1 / \delta A_i^2}
\]

\[
\delta A = \sqrt{\frac{1}{\sum_i 1 / \delta A_i^2}}
\]

Also weight nu bins by statistical uncertainty

• Sign convention
  – Changed definition of HWP from previous analysis to match sign at Delta with models
    • IHWP is "IN" ["OUT")\(^1\): +1 \([-1]\) \(^1\)"OUT" corresponds to a MySQL value of 0.
    • Target polarization greater [less] than 0: +1 \([-1]\)
    • LHRS (RHRS) transverse asymmetry: +1 \([-1]\)
Asym Accept. Cuts

Transverse LHRS

Transverse RHRS

Longitudinal LHRS

See Slides on 4/26/17 for the remaining good electron cuts used in my analysis
Scattering Angle Fit

- Weight reconstructed scattering angle by $1/\text{Mott}$ to get a better idea of central value of scattering angle and then fit it

$$\theta_{\text{rec}} = \exp(p_0 + p_1 E_p) + p_2 + p_3 E_p$$

<table>
<thead>
<tr>
<th>$E_0$ (MeV)</th>
<th>$p_0$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2254</td>
<td>3.485</td>
<td>-2.113</td>
<td>9.231</td>
<td>-0.871</td>
</tr>
<tr>
<td>3350</td>
<td>2.667</td>
<td>-1.723</td>
<td>9.124</td>
<td>-0.876</td>
</tr>
</tbody>
</table>

![Graph 1](image1.png)

![Graph 2](image2.png)
Scattering Angle Fit

- Can test $1/Mott$ weighting to reconstruction to comparing results at longitudinal setting to a fit to a constant
  - Mott weighting increases mean angle value by approx $0.5^\circ$
- Pointing angle is $5.77^\circ$
- Agreement within uncertainty (2% on rec. angle/0.04° on pointing result)
Out of Plane Polarization Angle

• Showed a bunch on this before and in my thesis it’s mostly math so...

• See for example slides on:
  – 01/11/17
  – 01/04/17
  – 11/30/16
  – 11/02/16
Dilution (3.3 GeV)

Use model Bosted dilution to fill in gaps of coverage
Tune Bosted to saGDH nitrogen data (see for example 04/01/15)
Blue band is systematic uncertainty of +/- 15%
Toby’s shown these before...
But updated comparison to tuned model

Dilution

2.2 GeV 5T Trans

2.2 GeV 5T Long
Reminder:
For asym comp. do a red. $\chi^2$ test
red. $\chi^2 = 1.3$ is ideal case
See David’s slides from 9/14/16

Showed comparison with Toby
on 04/26/17
2.2 GeV 5T Transverse

Asymmetry Comparison

Difference between LHRS and RHRS asymmetries is consistent with 0
Asymmetry Comparison

2.2 GeV 5T Transverse
2.2 GeV 5T Longitudinal

Asymmetry Comparison

Reduced $\chi^2 = 1.3608$

Apply Dilution

Corrected for Pb/Pt

LHRS
RHRS

MAID
Hall B
E08-027 Data

$W$ (MeV)
Cut Study

- Break up acceptance into different regions and therefore different scattering angles to check for bin-centering effects
- Do it at longitudinal because that has the best statistics
Cut Study

Loose/Tight/Hot cut is 5.77°/6.3°/4.9°
- Average of Tight and Hot is 5.6°±0.16°

Average asymmetry (Tight+Hot) is consistent with Loose asymmetry

Reduced $\chi^2 = 0.70$
Systematic Errors for Asym

- Dilution: 5-7% (15% for missing coverage)
- Beam polarization: 1.7%
- Target polarization: 2-4%
- Out-of-plane-polarization: 0.5-1.5%
- Pion contamination/asym negligible
  - From Melissa’s study and thesis
Polarized DS

- Using radiated Bosted model for now with following systematics:
  - UPDATED FIT FROM ERIC CHRISTY TO INCLUDE LOW Q2 SLAC DATA NOT USED IN ‘09 FIT!
  - 15% uncertainty on the model
  - Mott uncertainty 4-8% (as determined from 2% on scattering angle)
  - Radiative uncertainty 3%
- Add systematics in quadrature to get total model systematic

Comparing to data from HallC resonance database at kinematics similar to g2p

Error bars are STD of the average
Comparison of Old Fit and New Fit

Covering data from the grey band on the previous slide

$\theta = 1.5^\circ$

$\frac{d\sigma}{dE/d\Omega}$ (nb/MeV sr)

$W$ (GeV)

Continuation of Toby and I’s slides from 4/22/17
Comparison of Old Fit and New Fit

For $\theta = 1.5^\circ$

- SLAC ONEN1HAF, $E_0 = 11.799$ GeV, $Q^2 \approx 0.09$ GeV$^2$
- Bosted F1F209
- Christy16

Continuation of Toby and I’s slides from 4/22/17
Comparison of Old Fit and New Fit

\[ \theta = 1.5^\circ \]

Continuation of Toby and I’s slides from 4/22/17
Comparison of Old Fit and New Fit

\[ \vartheta = 4.0^\circ \]

Continuation of Toby and I’s slides from 4/22/17
Polarized DS

Before Tail Subtraction

Inner error bars are statistical and outer error bars are the total error of sys. added in quadrature with the stat.

Generated Christy16 model is inelastically radiated and includes unpolarized elastic tail

See my tech-note for radiation lengths used in the radiation calculation
Polarized Tail Subtraction

- Update code to latest J. Arrington FF’s
- Reproduces elastic cross-sections to ±3%
- Also big improvement over previous fits used by ROSETAIL in the low Q2 region
- Adjust stat error for removal of unwanted elastic events:
  \[
  \delta_{\text{sub}} = \left[ \left( \frac{1}{\delta_{\text{exp}}} \right)^2 \left( \frac{\sigma_{\text{sub}}}{\sigma_{\text{tail}} + \sigma_{\text{sub}}} \right) \right]^{\frac{1}{2}}
  \]

<table>
<thead>
<tr>
<th>Setting</th>
<th>(\delta_{\text{int}}) (%)</th>
<th>(\delta_{\text{ext}}) (%)</th>
<th>(\delta_{\text{FF}}) (%)</th>
<th>(\delta_{\theta}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3350 5T Tran.</td>
<td>1.0</td>
<td>1.5</td>
<td>3.0−5.0</td>
<td>3.5</td>
</tr>
<tr>
<td>2254 5T Tran.</td>
<td>1.0</td>
<td>1.5</td>
<td>3.0−5.0</td>
<td>3.5</td>
</tr>
<tr>
<td>2254 5T Long.</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 8.2: Systematic errors for the polarized radiative elastic tail calculation.
Tail-Sub
Polarized DS

- Not sure why tail-sub DS doesn’t go to zero
- Tail is ~20% too large (or DS 20% too small)
- I’ve checked and double-checked tail calc.
  - Uncertainties at 3% level
  - Comp MASCARAD/POLRAD asym/DS elastic calc with more experimentalist friendly formalism and they agree
    - See for example Andrew Puckett’s MIT thesis from 2010
  - Rules out any out-of-plane components diluting tail
- See similar thing if Toby does Asym*DataXS with no dilution correction
  - Could be scattering angle related??
  - Increase angle from 5.7 to 6.2 degrees and tail drops by ~20%
Inelastic Radiative Corrections

- Reminder: update unfolding procedure to deal with a changing scattering angle
  - Requires that extrapolation spectra have same angular dependence as the DS you’re trying to RC

![Graph showing theoretical systematic errors for polarized inelastic radiative corrections.](image)

Table 8.3: Theoretical systematic errors for the polarized inelastic radiative corrections.

<table>
<thead>
<tr>
<th>Setting</th>
<th>δ_{strag} (%)</th>
<th>δ_{soft} (%)</th>
<th>δ_{other} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3350 5T Tran.</td>
<td>&lt; 0.5</td>
<td>1.0–3.0</td>
<td>0.8</td>
</tr>
<tr>
<td>2254 5T Tran.</td>
<td>&lt; 1.0</td>
<td>1.0–2.5</td>
<td>0.8</td>
</tr>
<tr>
<td>2254 5T Long.</td>
<td>&lt; 1.0</td>
<td>1.0–2.5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

In general:
Theory systematic dominates at low W
Process systematic dominates at high W
Born Polarized DS

Error bars are total error: statistical error added in quadrature with the systematic

Small but noticeable difference between POLRAD and RADCOR so using POLRAD for internal going forward

Reminder: Tested systematic from doing RADCOR->POLRAD, POLRAD->RADCOR and RADCOR->POLRAD->RADCOR. Negligible effect
Extract the SSFs

- Because I’m using a model for the unmeasured component of the SSFs:

\[
g_1(x, Q^2) = K_1 \left[ \Delta \sigma_\parallel \left( 1 + \frac{1}{K_2} \tan \frac{\theta}{2} \right) \right] + \frac{2g_2(x, Q^2)}{K_2y} \tan \frac{\theta}{2} - \frac{g_1(x, Q^2)y}{2},
\]

\[
g_2(x, Q^2) = \frac{K_1y}{2} \left[ \Delta \sigma_\perp \left( K_2 + \tan \frac{\theta}{2} \right) \right] - \frac{g_1(x, Q^2)y}{2},
\]

\[
K_1 = \frac{MQ^2}{4\alpha} \frac{y}{(1-y)(2-y)},
\]

\[
K_2 = \frac{1 + (1-y)\cos\theta}{(1-y)\sin\theta}.
\]

Can compare MAID/Hall B model to g1 comp of g2 at the 5T kinematics to min. systematic
Evolving to Constant Q2

Use models to facilitate the extrapolation

\[ \delta_{\text{evolve}} = g_{1,2}^{\text{mod}}(x_{\text{data}}, Q_{\text{data}}^2) - g_{1,2}^{\text{mod}}(x_{\text{const}}, Q_{\text{const}}^2), \]

\[ x_{\text{const}} = Q_{\text{const}}^2 / (W^2 - M^2 + Q_{\text{const}}^2), \]

Change is only 7% (18%) at 3.3 GeV (2.2 GeV)

Decreases by 50% at long setting

Uncertainty in reconstructed Q2 is at 5% level based upon scat. angle uncertainty

Systematic is difference between using MAID and HALL B model and is applied linearly
Evolving Constant to Q2

- Procedure on g1 is easily tested with the Hall B data
- For g2 use our own data and try and evolve 2.2 GeV to 3.3 GeV

![Graph showing data points and fitting curves for g2 vs W (MeV)]
First Moment of $g_1$

For systematic error in all moments, assume the errors are fully correlated.

![Graph showing first moment of $g_1$](image)

**Table 8.4:** Results for the E08-027 $\Gamma_1(Q^2)$ integration.

<table>
<thead>
<tr>
<th>Setting</th>
<th>$\Gamma_1^{\text{meas.}}$</th>
<th>$\Gamma_1^{\text{low }x}$</th>
<th>$\Gamma_1^{\text{tot.}}$</th>
<th>$\delta_{\text{stat.}}$</th>
<th>$\delta_{\text{sys.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2254 5T Long.</td>
<td>-0.01539</td>
<td>0.0002</td>
<td>-0.01539</td>
<td>0.0006</td>
<td>0.0028</td>
</tr>
</tbody>
</table>
First Moment of $g_1$

**Graph Description:**
- **Upper Graph:**
  - Plot title: $\Gamma_1^p$ Integrand
  - X-axis: $W$ (MeV)
  - Y-axis: $\Gamma_1^p$ Integrand
  - Data points and error bars for Hall B, E08-027 ($Q^2 = 0.043$ GeV$^2$), and CLAS EG1b ($Q^2 = 0.0496$ GeV$^2$)

- **Lower Graph:**
  - Plot title: Cumulative Int.
  - X-axis: $W$ (MeV)
  - Y-axis: Cumulative Int. ($\times 10^{-2}$)
  - Cumulative line for Hall B, E08-027 ($Q^2 = 0.043$ GeV$^2$), and CLAS EG1b ($Q^2 = 0.0496$ GeV$^2$)
The calculated moments suggest a smooth and relatively flat approach to the low-momentum-transfer region. The extrapolated portions of the integral are not necessarily small and negligible. Their systematic uncertainties are not correlated in the same way as the statistical and experimental uncertainties. They are added linearly to the other systematic uncertainties. In the EG1b data, they are fully correlated.

The red star in Figure 8-27 represents the E08-027 data point, which is dominated by the Pascalutsa effect on the resultant integral. These parameters are discussed in Appendix E08-027.

The second order term gives the generalized longitudinal-transverse polarizability, further strengthening the relation between these integrated quantities and static properties of the nucleon. The original GDH sum rule is:

\[ I_A(Q^2 = 0) = -\frac{1}{4} \kappa_p^2 = -0.8010 \]

\[ I_{TT}(Q^2) = \frac{M^2}{4\pi^2\alpha} \int_{\nu_0}^{\infty} \frac{K(\nu', Q^2)\sigma_{TT}}{\nu'^2} d\nu' \]

\[ = \frac{2M^2}{Q^2} \int_0^{x_0} \left( g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right) dx . \]

Table 8.5: Results for the E08-027 \( I_A(Q^2) \) integration.
Extended GDH Sum

- Hall B
- Hall B ($g_2$ contribution)
- E08-027: $Q^2 = 0.043$ GeV$^2$
- CLAS EG1b: $Q^2 = 0.0496$ GeV$^2$
Forward Spin Polarizability

\[ \gamma_0(Q^2 = 0) = [-1.01 \pm 0.08 \text{ (stat)} \pm 0.10 \text{ (sys)}] \times 10^{-4} \text{ fm}^4 \]

Table 8.6: Results for the E08-027 $\gamma_0(Q^2)$ integration. Units are $10^{-4} \text{ fm}^4$. 

<table>
<thead>
<tr>
<th>Setting</th>
<th>$\gamma_0^{\text{meas.}}$</th>
<th>$\gamma_0^{\text{low }x}$</th>
<th>$\gamma_0^{\text{tot.}}$</th>
<th>$\delta_{\text{stat}}^{\text{tot.}}$</th>
<th>$\delta_{\text{sys}}^{\text{tot.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2254 5T Long.</td>
<td>-1.9337</td>
<td>-0.0007</td>
<td>-1.9337</td>
<td>0.1250</td>
<td>0.5710</td>
</tr>
</tbody>
</table>
Forward Spin Polarizability

\[ W (\text{MeV}) \]

\[ \gamma_0 \text{ Integr} \]

- Hall B
- Hall B (\(g_2\) contribution)
- E08-027 : \(Q^2 = 0.043 \text{ GeV}^2\)
- CLAS EG1b : \(Q^2 = 0.0496 \text{ GeV}^2\)
Table 8.6: Results for the E08-027 $\gamma_0^*(Q^2)$ integration. Units are $10^{-4}$ fm$^6$.

Table 8.7: Results for the E08-027 $\gamma_0^*(Q^2)$ integration. Units are $10^{-4}$ fm$^6$. 

\begin{equation}
\gamma_0^*(Q^2) = \frac{64M^4\alpha}{Q^{10}} \int_0^{x_0} x^4 \left( g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right) dx.
\end{equation}
Hyperfine Splitting (g1)

Relevant Integrals:

\[
\Delta_1 = \frac{9}{4} \int_0^\infty \frac{dQ^2}{Q^2} \left[ \left( \frac{G_M(Q^2) + G_E(Q^2)}{1 + \tau} \right)^2 + \frac{8m^2}{Q^2} B_1(Q^2) \right] \\
B_1(Q^2) = \int_0^{x_{\text{th}}} dx \beta_1(\tau) g_1(x, Q^2) \\
\beta_1(\tau) = \frac{4}{9} \left( -3\tau + 2\tau^2 + 2(2 - \tau) \sqrt{\tau(\tau + 1)} \right),
\]

Results from Carlson paper:

\[
\Delta_1 = 8.85 \pm 0.30 \text{ (stat)} \pm 3.57^{10} \text{ (sys)} \\
\Delta_1[0, Q^2_1] = \left( -\frac{3}{4} \kappa_{\mu}^2 r_p^2 + 18M^2 c_1 \right) Q^2_1
\]

Results adding in g2p data

<table>
<thead>
<tr>
<th>Term</th>
<th>(Q^2 \text{ (GeV}^2))</th>
<th>Contribution</th>
<th>Result</th>
<th>Stat</th>
<th>Sys</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta_1)</td>
<td>(0.0,0.043)</td>
<td>(F_2) and (g_1)</td>
<td>1.28</td>
<td>0.20</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>(0.043,5.0)</td>
<td>(F_2)</td>
<td>7.65</td>
<td>-</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(0.043,5.0)</td>
<td>(g_1)</td>
<td>-0.77</td>
<td>0.22</td>
<td>2.46</td>
</tr>
<tr>
<td></td>
<td>(5.0,\infty)</td>
<td>(F_2)</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(5.0,\infty)</td>
<td>(g_1)</td>
<td>0.45</td>
<td>-</td>
<td>0.45</td>
</tr>
<tr>
<td>Total (\Delta_1)</td>
<td></td>
<td></td>
<td>8.63</td>
<td>0.30</td>
<td>4.19</td>
</tr>
</tbody>
</table>
Hyperfine Splitting ($g_2$)

Compare Data/Models:

<table>
<thead>
<tr>
<th>Term</th>
<th>$Q^2$ (GeV$^2$)</th>
<th>MAID</th>
<th>Hall B</th>
<th>HB 2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_2$</td>
<td>(0.086,0.130)</td>
<td>-0.29</td>
<td>-0.30</td>
<td>-0.36</td>
</tr>
<tr>
<td></td>
<td>(0.05,0.20)</td>
<td>-0.16</td>
<td>-0.17</td>
<td>-0.26</td>
</tr>
<tr>
<td></td>
<td>(20,∞)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Total $\Delta_2$</td>
<td></td>
<td>-2.13</td>
<td>-1.96</td>
<td>-0.56</td>
</tr>
</tbody>
</table>

Table 8.9: Comparison of the $\Delta_2$ contribution to the hydrogen hyperfine splitting.

Compare Different Models:

$$B_2(Q^2) = \int_0^{x_{th}} dx \beta_2(\tau) g_2(x, Q^2)$$

$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)}.$$