Helium-4 Elastic Cross Section Comparison to Rosenbluth

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March 22, 2017

“Empty” runs normalized to experimental cross section

$$\frac{d\sigma^{raw}}{d\Omega dE'} = \frac{ps_1 N}{\frac{Q_{tot}}{e}\rho(LT)\varepsilon_{det}} \frac{1}{\Delta\Omega\Delta E'\Delta Z}$$

N is the yield, the number of scattered electrons

e is the electron charge

Quantitities from MySQL: ps_1 is the prescale factor, Q_{tot} is the total charge, ρ is the number density of the electron beam, LT is the livetime, ε_{det} is the product of all detector efficiencies.

$\Delta\Omega$ is the solid angle acceptance. Tight central cut used:

$-0.005 < \phi < 0.005$ | $-0.02 < \theta < 0.02$

$\Delta E'$ is the energy acceptance: 1 MeV bins

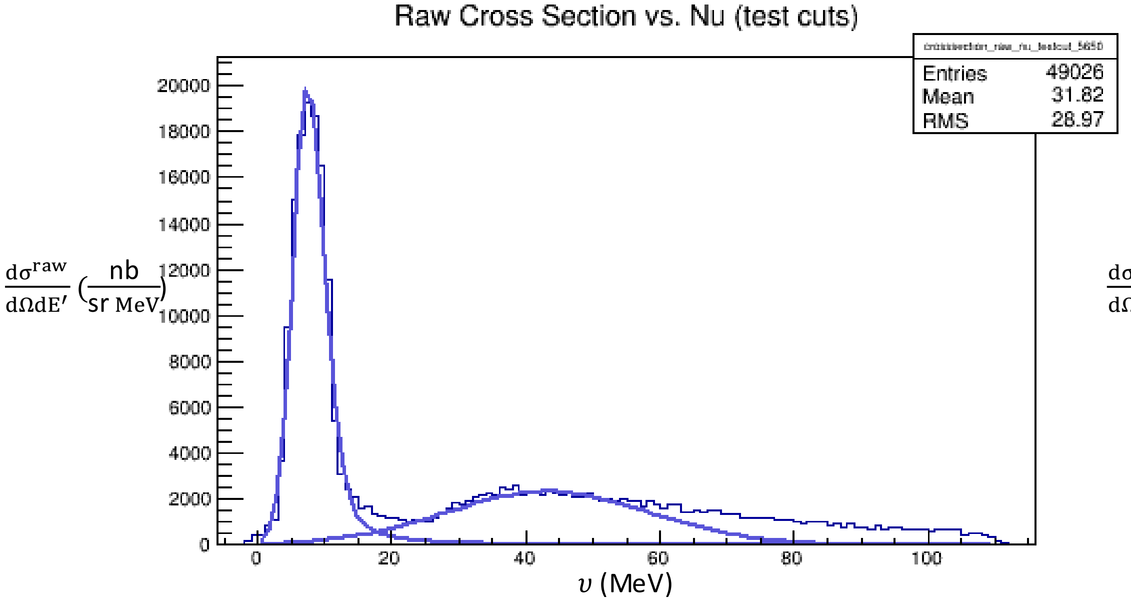
ΔZ is the target length from Ryan's Tech Note #19

All runs are 2.2 GeV incident energy, 5 T magnetic field, Helium-4 / ‘Empty’ target

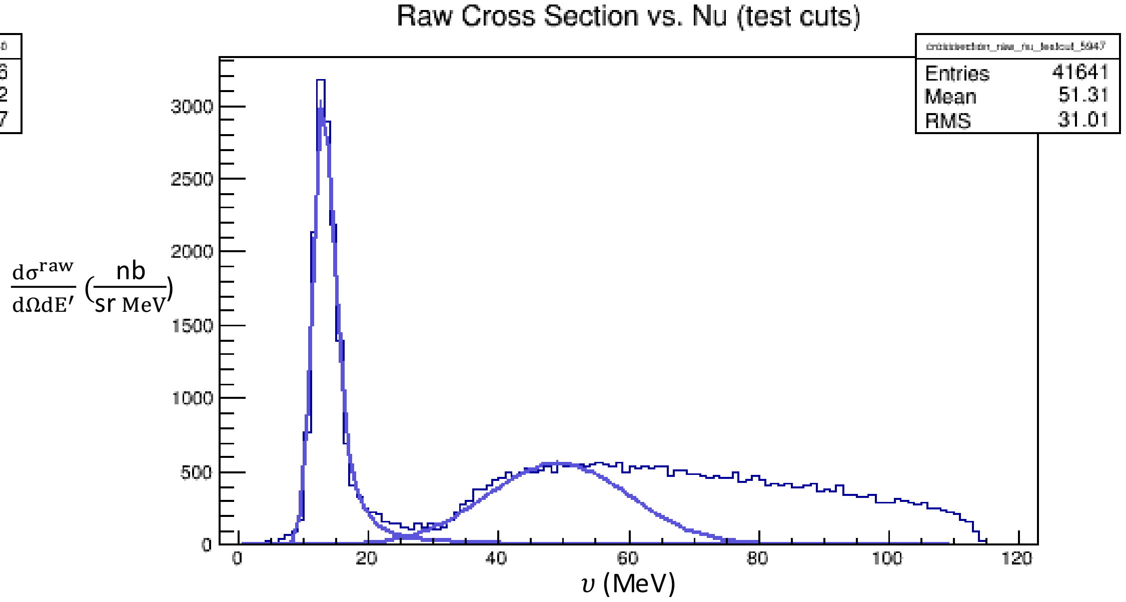
1. “The Spin Structure of 3He and the Neutron at Low Q^2, A Measurement of the Generalized GDH Integrand”, 116, V. Sulkosky, 2007
2. “Radiation Thickness, Collisional Thickness, and Most Probable Collisional Energy Loss for E08-027”, 17, R. Zielinski, 2016

Toby method used to fit elastic peak and remove quasielastic contamination

$Run\ 5650\ (Longitudinal)$



$Run\ 5947\ (Transverse)$



Fitting the elastic peak to a Gaussian-Landau convolution and the start of the quasielastic peak to a Gaussian gives a more accurate elastic peak

Sensitivity to fit bounds examined, varying the maximum elastic bound and minimum quasielastic bound by ± 5 MeV had minimal effect on integrated cross section.

Elastic peak integrated and radiative corrections applied

Mo and Tsai method used. Formula is lengthy, see image.
All quantities defined well in paper with the exception of “ΔE”

ΔE defined in separate Tsai paper as length of Landau tail on elastic peak, but there is some guesswork in the cut-off.
Average results:

$\frac{d\sigma}{d\Omega_{long}}$ = 183600 nb/sr | $\frac{d\sigma}{d\Omega_{trans}}$ = 22072 nb/sr

Fairly sensitive to choice of ΔE; “High” ΔE value chosen at very end of tail:

$\frac{d\sigma}{d\Omega_{long}}$ = 179474 nb/sr | $\frac{d\sigma}{d\Omega_{trans}}$ = 21575 nb/sr

“Low” ΔE value chosen at base of peak:

$\frac{d\sigma}{d\Omega_{long}}$ = 189581 nb/sr | $\frac{d\sigma}{d\Omega_{trans}}$ = 22791 nb/sr

Have not yet done uncertainty analysis on ΔE choice

$$
\begin{aligned}
\delta = \frac{-\alpha}{\pi}\Bigg(& \tfrac{28}{9} - \tfrac{13}{6}\ln\left(\frac{-q^2}{m^2}\right) + \left(\ln\frac{-q^2}{m^2} - 1 + 2Z\ln\eta\right)\left(2\ln\frac{E_1}{\Delta E} - 3\ln\eta\right) - \Phi\left(\frac{E_3 - E_1}{E_3}\right) - Z^2\ln\frac{E_4}{M} \\
& + Z^2\ln\frac{M}{\eta\Delta E}\left(\frac{1}{\beta_4}\ln\frac{1+\beta_4}{1-\beta_4} - 2\right) + \frac{Z^2}{\beta_4}\left\{\tfrac{1}{2}\ln\frac{1+\beta_4}{1-\beta_4}\ln\frac{E_4+M}{2M} - \Phi\left[-\left(\frac{E_4-M}{E_4+M}\right)^{1/2}\left(\frac{1+\beta_4}{1-\beta_4}\right)^{1/2}\right]\right\} \\
& + Z\left[\Phi\left(-\frac{M-E_3}{E_1}\right) - \Phi\left(\frac{M(M-E_3)}{2E_3E_4 - ME_1}\right) + \Phi\left(\frac{2E_3(M-E_3)}{2E_3E_4 - ME_1}\right) + \ln\left|\frac{2E_3E_4 - ME_1}{E_1(M-2E_3)}\right|\ln\left(\frac{M}{2E_3}\right)\right] \\
& - Z\left[\Phi\left(-\frac{E_4-E_3}{E_3}\right) - \Phi\left(\frac{M(E_4-E_3)}{2E_1E_4 - ME_3}\right) + \Phi\left(\frac{2E_1(E_4-E_3)}{2E_1E_4 - ME_3}\right) + \ln\left|\frac{2E_1E_4 - ME_3}{E_3(M-2E_1)}\right|\ln\left(\frac{M}{2E_1}\right)\right] \\
& - Z\left[\Phi\left(-\frac{M-E_1}{E_1}\right) - \Phi\left(\frac{M-E_1}{E_1}\right) + \Phi\left(\frac{2(M-E_1)}{M}\right) + \ln\left|\frac{M}{2E_1 - M}\right|\ln\left(\frac{M}{2E_1}\right)\right] \\
& + Z\left[\Phi\left(-\frac{M-E_3}{E_3}\right) - \Phi\left(\frac{M-E_3}{E_3}\right) + \Phi\left(\frac{2(M-E_3)}{M}\right) + \ln\left|\frac{M}{2E_3 - M}\right|\ln\left(\frac{M}{2E_3}\right)\right]\Bigg) \\
& - \frac{\alpha}{\pi}\left(-\Phi\left(\frac{E_1-E_3}{E_1}\right) + \frac{Z^2}{\beta_4}\left\{\Phi\left[\left(\frac{E_4-M}{E_4+M}\right)^{1/2}\left(\frac{1-\beta_4}{1+\beta_4}\right)^{1/2}\right] - \Phi\left[\left(\frac{E_4-M}{E_4+M}\right)^{1/2}\right] + \Phi\left[-\left(\frac{E_4-M}{E_4+M}\right)^{1/2}\right]\right\}\right). \quad \text{(II.6)}
\end{aligned}
$$

$$
\begin{aligned}
\delta_t = -\{ & [b_w t_{iw} + \tfrac{1}{2}bT]\ln(E_1/\eta^2\Delta E) \\
& + [b_w t_{fw} + \tfrac{1}{2}bT]\ln(E_3/\Delta E)\}; \quad \text{(II.9)}
\end{aligned}
$$

$$
d\sigma/d\Omega|_{\text{meas}} = d\sigma/d\Omega|_{\text{Rosenbluth}}\exp(\delta + \delta_t),
$$

1. “Radiative Corrections to Elastic and Inelastic ep and up Scattering”, L. Mo and Y. Tsai, *Reviews of Modern Physics 41:1*, 1969
2. “Radiative Corrections to Electron-Proton Scattering”, Y. Tsai, *Physical Review 122:6*, 1961

Compared to Rosenbluth Cross Section for Electron Scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \times \{\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\}$$

Helium form factors taken from McCarthy, Sick, and Whitney, where $G_E = 2F_{ch}$

Longitudinal Results:

$\frac{d\sigma}{d\Omega_{exp}}$ = 183600 nb/sr

$\frac{d\sigma}{d\Omega_{Rosenbluth}}$ = 172449 nb/sr

6.4% Difference

"High" ΔE: 4.1% Difference
"Low" ΔE: 9.9% Difference

Transverse Results:

$\frac{d\sigma}{d\Omega_{exp}}$ = 22072 nb/sr

$\frac{d\sigma}{d\Omega_{Rosenbluth}}$ = 25784 nb/sr

-14.4% Difference

"High" ΔE: -16.3% Difference
"Low" ΔE: -11.6% Difference

1. "Spin Structure of 3He and the Neutron at Low Q^2; A Measurement of the Extended GDH Integral and the Burkhardt-Cottingham Sum Rule", K. Slifer, 2004
2. "Electromagnetic Structure of the Helium Isotopes", J. McCarthy, I. Sick, R. Whitney, *Phys. Rev. C 1396:15*, 1977

Conclusions

- Empty runs compare well to Rosenbluth elastic cross section, but are sensitive to choice of ΔE factor from Mo and Tsai radiative corrections
- Still need to complete uncertainty analysis on ΔE correction and Rosenbluth calculation from the reconstructed scattering angle
- If choice of ΔE is good, there may be a small correction needed to make the cross sections match