The Generalized GDH Sum Rule

Measuring the Spin Structure of He-3 and the Neutron using Nearly Real Photons

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GDH Sum Rule \((Q^2 = 0)\)

For circularly polarized real photons \((Q^2 = 0)\):

\[
I_{\text{GDH}} = \int_{\nu_0}^{\infty} \left[ \sigma_{\frac{1}{2}}(\nu) - \sigma_{\frac{3}{2}}(\nu) \right] \frac{d\nu}{\nu} = -2\pi^2 \alpha \left( \frac{\kappa}{M} \right)^2
\]

\[
I_{\text{GDH}}^n = -233 \ \mu b \ \& \ I_{\text{GDH}}^{3\text{He}} = -498 \ \mu b
\]

This sum rule relates the real photoabsorption cross section difference to the anomalous part of the target magnetic moment \(\kappa\).

Causality \(\rightarrow\) Dispersion Relation
Unitarity \(\rightarrow\) Optical Theorem
Lorentz & Gauge Invariance \(\rightarrow\)
\(\rightarrow\) Low Energy Theorem
Generalized Integral for $S = 1/2$

When the integrand is generalized to $Q^2 > 0$:

$$I = \int_{\nu_0}^{\infty} \left[ \frac{K(\nu, Q^2)}{\nu} \right] \left[ \sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2) \right] \frac{d\nu}{\nu}$$

$$K(\nu, 0) = \nu$$

...the integral can form a sum rule proportional to the virtual photon Compton Amplitude $S_1(\nu, Q^2)$ [see for example: X. Ji & J. Osbourne J. Phys. G: Nucl. Part. Phys. 27, 127 (2001)], which can be calculated over the full $Q^2$ range using different theoretical tools.
GDH Integral for $Q^2 > 0.1 \text{ GeV}^2$

1. At high $Q^2$, the integral is very close to zero. (HERMES [Eur. Phys. J. C26, 527 (2003)])

2. At intermediate $Q^2$, the integral drops dramatically. (JLAB [PRL 89, 242301 (2002)])

3. At low $Q^2$, the integral must “turn over” in order to satisfy the sum rule.
A polarized $^3$He nucleus “stands in” as a polarized neutron. Detected only the scattered electron at 6° and 9° using the right septum magnet and the standard Hall A HRS package. $^3$He target cells were specifically designed and constructed to minimize radiative corrections. We have data for both longitudinal (parallel) and transverse (perpendicular) target polarizations. Contamination from the glass and Nitrogen are subtracted using data from reference cell runs for each kinematic. Measured “double” polarized cross sections and asymmetries for inclusive electron scattering from a polarized $^3$He target.
Polarized Inclusive Electron Scattering

Energy Lost by Incident Electron:
\[ \nu = E - E' \]

4-Momentum Transferred:
\[ Q^2 = -q^2 \approx 4EE' \sin^2 \left( \frac{\theta}{2} \right) \]

Invariant Mass of the Hadron Decay Products:
\[ W_X = |p + q| = \sqrt{M_N^2 + 2\nu M_N - Q^2} \]
Kinematic Coverage
Experimental Observables

The measured cross section differences are:

\[ \Delta \sigma_{\parallel} = \frac{4\alpha^2}{MQ^2} \frac{E'}{E} \left[ \left( \frac{E + E' \cos(\theta)}{\nu} \right) g_1 - \left( \frac{Q^2}{\nu^2} \right) g_2 \right] \]

\[ \Delta \sigma_{\perp} = \frac{4\alpha^2}{MQ^2} \frac{E'}{E} \left( \frac{E' \sin(\theta)}{\nu} \right) \left[ g_1 + \left( \frac{2E}{\nu} \right) g_2 \right] \]

The GDH Integrand is given by:

\[ \sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}} = -2\sigma'_{TT} = \frac{8\pi^2\alpha}{MK} \left[ g_1 - \left( \frac{Q^2}{\nu^2} \right) g_2 \right] \]

To access the GDH integrand for \( Q^2 > 0 \), we need a longitudinally and transversely polarized target.
Preliminary Results

Non-convention dependant part of GDH integrand
Non-convention dependant part of GDH integrand
Preliminary Results

Non-convention dependant part of GDH integrand
Summary: Expected Results

This data set complements the E94010 data set below $Q^2 = 0.10$ GeV$^2$ with improved precision.

1. Turnover?
2. Slope at low $Q^2$?
3. Extrapolation to the real photon point ($Q^2 = 0$)?

In addition, we will also extract the moments of the spin structure functions and forward spin polarizabilities.
Collaboration List


Spokesperson

Thesis Student
Effective Polarized Neutron Target

Largest contribution to the $^3\text{He}_3$ wave function is a neutron and two antialigned protons [J.L. Friar et al, Phys. Rev. C42, 2310 (1990)]:

\[ n \uparrow \quad p \quad p \quad S = 1/2 \]

\[ S = 0.90 \quad D \approx 0.08 \quad S' \approx 0.02 \]

Traditionally neutron quantities have been extracted from $^3\text{He}_3$ quantities using the “effective polarization” prescription following C. Ciofi degli Atti & S. Scopetta [Phys. Lett. B 404, 223 (1997)], for example:

\[ I^n(Q^2) = \frac{1}{p_n} \left[ I^3(Q^2)_{\text{helium-3}} - 2p_pI^p(Q^2)_{\text{proton}} \right] \]
Preliminary Results

Spin Structure Functions weighted by $x$
Spin Structure Functions weighted by $x$
Preliminary Results

Spin Structure Functions weighted by $x$
Preliminary Results

2.135 GeV, 6°

Preliminary

2.845 GeV, 6°

Preliminary
Preliminary Results

2.234 GeV, 9°

\[ \sigma_{LT} \]

3.319 GeV, 9°

\[ \sigma_{LT} \]

Preliminary
Preliminary Results

3.775 GeV, $g^\prime$

$\sigma_{LT}$

4.404 GeV, $g^\prime$

$\sigma_{LT}$

Preliminary

Preliminary