Origin of nucleon spin has been investigated extensively after the EMC discovery that almost none of nucleon spin is carried by quarks [1]. Recent studies of polarized parton distribution functions (PDFs) are found in Ref. [2]. Although a gluon-spin contribution is not determined accurately, orbital angular momenta are likely to be the crucial quantities in explaining the nucleon spin. Such contributions have been investigated recently by generalized parton distribution functions in lepton scattering [3] and will be studied possibly at hadron facilities [4].

There are other quantities which are sensitive to the orbital angular momenta. For example, there are twist-two structure functions \( b_1 \) and \( b_2 \) in spin-one hadrons [5,6]. They could be related to the orbital angular momenta of internal constituents because they vanish if the constituents are in the S wave. Of course, they probe a different aspect from current studies for the nucleon-spin issue in the parton level. Twist-two structure functions were introduced in describing lepton deep inelastic scattering \([7]\). A useful sum rule for the twist-two function \( b_1 \) was proposed in Ref. [8], and it is expressed in terms of tensor-polarized structure function for the deuteron, \( W_{\mu \nu} [6,18] \). It is intended to understand the current status of the tensor distributions. Obtained distributions could be used for proposing future experiments.

The purpose of this research is to propose a simple parametrization for the tensor-polarized quark and antiquark distribution functions by analyzing the HERMES data. It is intended to understand the current status of the tensor distributions. Obtained distributions could be used for comparing them with theoretical model calculations and for proposing future experiments.

The structure function \( b_1 \) is defined in the hadron tensor \( W_{\mu \nu} \) [6,18]. It is expressed in terms of tensor-polarized distributions (\( \delta T q \) and \( \delta T \bar{q} \)) as [6,8,20]

\[
b_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2 \left[ \delta T q_i(x, Q^2) + \delta T \bar{q}_i(x, Q^2) \right],
\]

where \( i \) indicates the flavor of a quark and \( e_i \) is the charge of the quark. The variables \( Q^2 \) and \( x \) are defined by the momentum transfer \( q \) as \( Q^2 = -q^2 \) and \( x = Q^2/(2M_N \nu) \), where \( M_N \) and \( \nu \) are the nucleon mass and the energy transfer, respectively. Hereafter, the \( Q^2 \) dependence is not explicitly written in the PDFs. In this work, the \( b_1, \delta T q_i, \delta T \bar{q}_i \), and unpolarized PDFs are defined by the ones per nucleon for a nuclear target, namely, they are divided by the factor of 2 if it is the deuteron. The tensor-polarized distribution \( \delta T q \) is defined by

\[
\delta T q_i(x) = q_i^0(x) - \frac{q_i^{+1}(x) + q_i^{-1}(x)}{2},
\]

where \( q_i^\pm \) indicates an unpolarized-quark distribution in the hadron spin state \( \lambda \), and it is also defined by the distribution per nucleon. Namely, \( \delta T q \) indicates an unpolarized-quark
distribution in a tensor-polarized spin-one hadron. It should be noted that the notation $\delta_T q$ is not the transversity distribution, for which similar notations ($\delta$ or $\Delta_T$) are used in nucleon-spin studies, throughout this article. A sum rule exists for $b_1$ in a parton model [8]:

$$\int dx b_1(x) = -\frac{5}{24} \lim_{t \to 0} F_Q(t) = 0,$$

(3)

if the tensor-polarized antiquark distributions vanish $\int dx \delta_T \bar{q}(x) = 0$. Here, $F_Q(t)$ is the electric quadrupole form factor of a spin-one hadron at the momentum squared $t$.

We analyze the HERMES $b_1$ data for the deuteron. The tensor-polarized distributions are introduced as the unpolaraized PDFs in the deuteron ($D$) multiplied by a common weight function $\delta_T w(x)$:

$$\delta_T \bar{q}^{D}(x) = \delta_T w(x) \bar{q}^{D}(x),$$

$$\delta_T q^{D}(x) = \alpha_q \delta_T w(x) q^{D}(x).$$

(4)

Namely, certain fractions of quark and antiquark distributions are tensor polarized and such probabilities are given by the function $\delta_T w(x)$ and an additional constant $\alpha_q$ for antiquarks in comparison with the quark polarization. The $x$ dependence of $\delta_T w(x)$ for antiquarks could be different in general from the one for quarks. However, it is not the stage of suggesting such a difference from experimental measurements as it will become obvious later in this article.

It is known that nuclear modifications are less than a few percent for the unpolarized PDFs in the deuteron [21]. The tensor-polarized distributions cannot be determined within a few percent accuracy at this stage. Therefore, the nuclear tensor-polarized distributions cannot be determined within a few percent accuracy at this stage. W

The unpolarized PDFs $u_n(x), d_n(x), \cdots, s(x)$ could be taken from a recent global analysis, for example, by CTEQ [22], GJR [23], or MSTW [24]. In this work, the LO version of the MSTW parametrization is used. For the functional form of $\delta_T w(x)$, we note that there is a constraint from the sum rule in Eq. (3). The integrated tensor polarization, namely, the first moment, should vanish for the valence quarks. It indicates that there should be a node in the $x$-dependent function, so that an appropriate parametrization could be

$$\delta_T w(x) = ax^b(1 - x)^c(x_0 - x),$$

(7)

where $x_0$ is the position where $\delta_T q_n(x)$ and $\delta_T \bar{q}(x)$ vanish. If the first moments vanish for the valence-quark distributions, the constant $x_0$ is expressed by the other parameters as

$$x_0 = \int_0^1 dx x^{b+1}(1 - x)^c u_n(x) + d_n(x) \right) \int_0^1 dx x^b(1 - x)^c u_n(x) + d_n(x).$$

(8)

The $a, b, c,$ and $\alpha_q$ are the parameters to be determined from experimental measurements.

The parametrization of Eq. (7) is motivated by the following considerations. First, the parton model indicates the existence of a node as mentioned. Next, we expect to have smooth polynomial functional forms in the limits, $x \to 0$ and 1, as usual in unpolarized and longitudinally-polarized PDFs. In addition, the existence of the node and the functional form are, for example, supported by theoretical estimates of a convolution model, or so-called binding model, where the $D$-state admixture gives rise to an $x$-distribution with a node in $b_1$ including the tensor-polarized antiquark distributions [5,6,9].

From an analysis of the HERMES experimental data, the optimum function $\delta_T w(x)$ and $\alpha_q$ are determined. It is obvious from the data that the $b_1$ structure functions are not accurately measured to discuss scaling violation or even details of $x$ dependence. Therefore, a simplification is made by ignoring the scaling violation. The $Q^2$ value is fixed at $Q^2 = 2.5$ GeV$^2$, which is about the average $Q^2$ of the HERMES measurements, for calculating the unpolarized PDFs [24] in Eqs. (5) and (6). We made two types of analyses:

(i) Set 1: Tensor-polarized antiquark distributions are terminated ($\alpha_q = 0$);
that the parameter \( \alpha_q \) is estimated, we obtain

\[
\int dx_b(x) = -\frac{5}{24} \lim_{t \to 0} F_Q(t) + \frac{1}{18} \int dx [8 \delta_T \bar{u}(x) + 2 \delta_T d(x) + \delta_T s(x) + \delta_T \bar{s}(x)] = 0.0058.
\]

The choice of parametrization of Eq. (7) for the antiquark distributions could affect the numerical result. However, as it is obvious from Figs. 1 and 2, the antiquarks contribute only at small \( x \) (\( x < 0.1 \)). As long as the function \( \delta_T w(x) \) is a smooth function at \( x < 0.1 \), the result is not significantly changed.

This work is the first attempt to parametrize the tensor-polarized valence-quark and antiquark distributions. Including the antiquark tensor polarization, we obtained much smaller \( \chi^2 / \text{d.o.f.} \) and it led to a finite sum as shown in Eq. (9). This is a new and interesting result which needs to be explained theoretically. The integral is compared with...
the HERMES results [19], \( \int_{0.002}^{0.085} dx b_1(x) = [1.05 \pm 0.34(\text{stat}) \pm 0.35(\text{sys})] \times 10^{-2} \) and \( \int_{0.002}^{0.085} dx b_1(x) = [0.35 \pm 0.10(\text{stat}) \pm 0.18(\text{sys})] \times 10^{-2} \) in the restricted range with \( Q^2 > 1 \text{ GeV}^2 \). The integral of Eq. (9) is similar to the Gottfried sum [25]

\[
\int dx \left[ F_T^p(x) - F_T^n(x) \right] = \frac{1}{3} + \frac{2}{3} \int \left[ \bar{u}(x) - \bar{d}(x) \right],
\]

where the deviation from \( \int[u_1(x) - d_1(x)]/3 = 1/3 \) indicates flavor asymmetric antiquark distributions. In the \( b_1 \) case, the finite sum \( \int dx b_1 \) suggests that a finite tensor-polarized antiquark distribution should exist.

It is obvious from Fig. 1 that much better measurements are needed to investigate the details of tensor-polarized distributions particularly at medium and large \( x (>0.2) \). Such measurements could be possible, for example, at JLab (Thomas Jefferson National Accelerator Facility) by measuring \( b_1 \) and also at hadron facilities such as J-PARC (Japan Proton Accelerator Research Complex) [26] and GSI-FAIR (Gesellschaft für Schwerionenforschung—Facility for Antiproton and Ion Research) [27] by Drell-Yan processes with a polarized deuteron [12]. In particular, the Drell-Yan processes are suitable for directly finding the tensor-polarized antiquark distributions in Eq. (9).

**Summary:** In this work, optimum tensor-polarized quark and antiquark distributions are proposed from the analyses of HERMES data on \( b_1 \) for the deuteron. We found that a significant tensor quark polarization exists if the overall tensor polarization vanishes for the valence quarks although such a result could depend on the assumed functional form. Further experimental measurements are needed for \( b_1 \) such as at JLab as well as Drell-Yan measurements with the tensor-polarized deuteron at hadron facilities, J-PARC and GSI-FAIR. On the other hand, it is interesting to conjecture a possible physics mechanism to create a finite tensor-polarized antiquark distribution.

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[20] The overall factor 1/2 is introduced in \( b_1 \) as usual in defining \( F_1 \) and \( g_1 \) in terms of PDFs.


